UCLA Department of Economics **Fall 2015**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and each part is worth 10 points. Parts 1 and 2 have one question each, and Part 3 has two questions worth 5 points each. All questions are required. **Answer each question in a separate bluebook.**

You should turn in (at least) FOUR bluebooks, one (or more, if needed) bluebook(s) for each question.

Part 1

Consider a stochastic growth economy with one representative agent and three production sectors—two market sectors and one nonmarket (home production) sector. Each of two market sectors employs labor and two types of capital: equipment and structures. Denote these by H, K_E and K_S. Sector 1, the consumption good sector, produces a market consumption good, C_M, and structures using the technology

(*) $$C_{M,t} + K_{S,t+1} = Y_{1,t} = z_t K_{E,1,t}^{\theta_1} K_{S,1,t}^{\theta_2} H_{1,t}^{1-\theta_1-\theta_2} + (1-\delta_S) K_{S,t}.$$

Here, z_t is technology shock where $\log z_{t+1} = \rho_z \log z_t + \varepsilon_{1,t+1}$, $\varepsilon_1 \sim N(0, \sigma_1^2)$.

The second sector, sector 2, uses the same inputs to produce equipment and consumer durables, D_t. The technology and resource constraint for this sector is

(**) $$D_{t+1} + K_{E,t+1} = Y_{2,t} = q_t z_t K_{E,2,t}^{\theta_1} K_{S,2,t}^{\theta_2} H_{2,t}^{1-\theta_1-\theta_2} + (1-\delta_E) K_{E,t} + (1-\delta_D) D_t.$$

In this sector, $q_t z_t$ is the technology shock and $\log q_{t+1} = \rho_q \log q_t + \varepsilon_{2,t+1}$, $\varepsilon_2 \sim N(0, \sigma_2^2)$. Notice that the Cobb-Douglas production function combining capital and labor are identical in the two sectors.

In the home, households combine consumer durables and labor (L) to produce a nonmarket consumption good, C_N. In particular, $C_{N,t} = F(D_t, L_t)$, where F is a constant returns to scale production function. Households have one unit of time to divide between market work, home work, and leisure. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \log C_{M,t} + (1-\alpha) \log C_{N,t} + A \log(1 - H_t - L_t) \right].$$

A. Carefully formulate the dynamic program that would be solved by a social planner in this economy. Be sure to be clear about the state variables and choice variables.

B. Derive expressions that determine how the planner allocates a given amount of capital and labor across the two market sectors. Prove that the same fraction of each input is allocated to a given sector in period *t*. That is, show that $H_{1,t} = \phi_t H_t$, $K_{E,1,t} = \phi_t K_{E,t}$ and $K_{S,1,t} = \phi_t K_{S,t}$. Obviously the remainder, a fraction $(1-\phi_t)$, is allocated to sector 2.

C. Show that the result obtained in part B can be used to aggregate the resource constraints (*) and (**) into one resource constraint (derive it). Repeat part A given this result.

D. Define a *recursive competitive equilibrium* for this economy. Be sure there are markets for both market goods. What is the relative price of the output of sector 2? Explain.

E. Suppose an empirical fact about the world is that as durable goods become cheaper, households spend less time in nonmarket activities (female labor supply increases, for example). If you wanted this model to be consistent with this fact, what would this mean for the functional form you would choose for the home production technology, F? (Note: This question is not so much asking for a functional form for F, although that would be fine, but is asking for list of properties that this function should possess.)

Part 2

Time is discrete and runs from zero to infinity, $t \in \{0, 1, \ldots\}$. Every period a stochastic state is drawn from the finite set $\{1, 2, \ldots, S\}$. The initial state is fixed and denoted by s_0. The history of states up to time t is denoted by $s^t \equiv (s_0, s_1, \ldots, s_t)$. The probability of history s^t is denoted by $\pi(s^t)$.

There are $K \geq S$ Lucas trees in the economy. Tree $k \in \{1, \ldots, K\}$ pays dividend $d_k(s_t)$ at time t after history s^t. Notice that the dividend only depend on the *current* state. The aggregate endowment is $y(s_t) = \sum_{k=1}^{K} d_k(s_t)$, the sum of all dividends of all trees. We assume that the *dividend matrix*

$$D \equiv \begin{bmatrix} d_1(1), \ldots, d_K(1) \\ d_1(2), \ldots, d_K(2) \\ \vdots \\ d_1(S), \ldots, d_K(S), \end{bmatrix}$$

has rank S.

The economy is populated by I agents denoted by $i \in \{1, \ldots, I\}$. Agent i derives the inter-temporal utility:

$$\sum_{t, s^t} \beta^t \pi(s^t) u_i \left[c_{it}(s^t) \right],$$

over state-contingent consumption plans, where $u_i(c)$ satisfies standard conditions: it is concave, bounded, twice continuously differentiable, and satisfies $\lim_{c \to 0} u'(c) = \infty$. Notice that all agents have identical time discount factors, identical beliefs, and only differ in terms of their period utility function, $u_i(c)$. Agent i is endowed at time $t = 0$ with some shares of the trees which we denote by n_{ik0}. Of course, we must have that $\sum_{i=1}^{I} n_{ik0} = 1$ for each k.

1. (2pt) Assume first that complete markets open only at time zero, in which agents can trade state-contingent consumption claims: that is, they can buy consumption to be delivered at time t after history s^t, for all times and histories.

   (a) (1pt) Define an allocation; define a price system; define an equilibrium.

   (b) (1pt) Show that in an equilibrium, the consumption of any agent $i \in \{1, \ldots, I\}$ at (t, s^t) is history independent: it only depends on the the current state, s_t, and so can be written $c_i(s_t)$.

2. (2pt) Next consider the same environment with sequential markets. Precisely, assume that at each (t, s^t), agents can trade the K trees as well as a complete set of one-step ahead Arrow securities, that is, securities that promise to pay one unit of consumption good next period if the realized state is s_{t+1}.

   (a) (1pt) Define an allocation; define a price system; define an equilibrium.

   (b) (1pt) Derive the equilibrium relationship between the price of the trees, the dividend of the trees, and the price of one-step ahead Arrow securities.

3. (6pt) Given any time-zero market equilibrium consider, as we did in class, the corresponding sequential-market equilibrium with the same consumption allocation.

   (a) (1pt) Derive the relationship between the price of state-contingent consumption claims in the time-zero market equilibrium, and the price of one-step ahead Arrow securities in the corresponding sequential markets equilibrium.

   (b) (5pt) Consider the problem of implementing the equilibrium consumption plan in sequential markets.

      i. (2pt) Show that this can be done by trading trees *only at time zero*, and without trading any Arrow-Debreu securities.

      ii. (1pt) Assuming that $K = S$, derive a formula for the portfolio of trees that agent i must hold at time zero. Show that, with these portfolios, the market for trees clear.

      iii. (1pt) Suppose that, in addition to their initial tree endowment, agents also receive a labor income endowment $e_i(s_t)$ every period. Is it still possible to implement the consumption plan by trading trees only at time zero? Why or why not?

      iv. (1pt) Suppose that agents have heterogenous beliefs. Is it still possible to implement the consumption plan by trading trees only at time zero? Why or why not?

Part 3

Question 1

Consider an overlapping generations growth model in which output is given by $GDP = K_t^{\alpha} L_t^{1-\alpha}$ and in which, at each period $t = 1, 2, 3, \ldots$ a new cohort of two-period lived agents of measure one is born each endowed with one unit of labor time that they supply inelastically. Assume these agents have preferences $\log c_t^y + \beta \log c_{t+1}^o$. Assume that in steady-state, the government chooses a constant tax rate on labor income τ to finance the interest payments due on a constant stock of government debt B.

- **5 points** Present the equations characterizing the steady-state of this economy. These include the government budget constraint, the transition equation for the physical capital stock, the savings decision of each young cohort, and the first order conditions that determine the equilibrium interest rate and wage. You should be able to reduce these equations to three equations in three unknowns (the steady-state tax rate τ, the steady-state interest rate R, and the steady-state capital stock K) as a function of the steady-state value of government debt.

- **5 points** Show that in steady-state, government debt crowds out private capital. Specifically argue that in steady-states with higher outstanding government debt, the steady-state capital stock K and steady-state output are lower. Be careful to reason through what happens to the steady state tax rate τ, the steady state wage, and the steady state interest rate.

Part 3

Question 2 Heterogeneous Consumers, Growth, and Superior Goods

The following economy has a unit mass of households. Half of the households have the following preferences:

$$\max \sum \beta^t \{\ln(m_t) + \phi \ln(1 - h_t)\} \tag{1}$$

$$\tag{2}$$

These type-1 households maximize utility subject to the following budget constraint

$$w_t h_t \geq m_t \tag{3}$$

The other households have the following preferences:

$$\max \sum \beta^t \{\alpha \ln(m_t) + (1 - \alpha) \ln(s_t + \bar{s}) + \phi \ln(1 - h_t)\} \tag{4}$$

These type-2 households maximize utility subject to the following budget constraint:

$$w_t h_t \geq m_t + p_t s_t \tag{5}$$

There is a competitive representative firm that produces the "m" good:

$$\max\{A_t h_{mt} - w_t h_{mt}\} \tag{6}$$

There is a competitive representative firm that produces the "s" good:

$$\max\{p_t A_t h_{st} - w_t h_{st}\} \tag{7}$$

Technology, A_t, evolves as follows:

$$A_t = \exp(\gamma t) \tag{8}$$

(1) Define a competitive equilibrium for this economy.

(2) Suppose that $\bar{s} = 0$. Solve for formulae that can be used to solve for allocations and relative prices at any date. Does the value of α impact the quantity of labor chosen by either household? Does the value of α impact the relative price of s? Why is the value of h for either household independent of the level of technology

(3) Suppose that $\bar{s} > 0$. Draw 3 separate graphs as follows. Do not worry about solving for a closed-form solution. The graphs should show *approximate* patterns which follow from the equations that characterize the equilibrium. Time is on the horizontal axis for each graph. For the vertical axis, graph 1

shows hours worked for the type 2 household, graph 2 shows consumption of the s good for the type 2 household, and graph 3 shows consumption of the m good for the type 2 household. Discuss the economic forces impacting the variables in each of the graphs. Finally, discuss why the solution to the type 1 household problem is not impacted by the inclusion of the s good in the problem.