UCLA Department of Economics **Fall 2013**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider a problem solved by a planner who maximizes the following:

$$E\sum_{t=0}^{\infty}\beta^t U(c_t, 1-h_t).$$

Here, c_t is consumption, h_t is hours worked, the function U is assumed to have all the usual properties, and $0 < \beta < 1$. Output is produced according to a constant returns to scale technology, $y_t = z_t F(k_t, h_t)$, where y_t is output and k_t is the stock of capital. The variable z_t is a technology shock that evolves through time according to a first order autoregressive process with an unconditional mean of 1 and unconditional variance of σ_z^2. The stock of capital is assumed to depreciate at the rate δ each period.

Output can be used for consumption, investment (i_t) or government purchases (g_t). Investment in period t becomes productive capital one period later, $k_{t+1} = (1-\delta)k_t + i_t$. Government spending is an exogenous random variable that, like the technology shock, follows a first order autoregressive process, in this case with an unconditional mean of $\bar{g}$ and unconditional variance of σ_g^2. Innovations to this process are assumed to be independent of innovations to the technology shock process. In addition, government purchases are financed with lump sum taxes. Assume, initially, that government purchases do not directly affect preferences or the technology; they are simply thrown into the sea.

(a) Suggest functional forms for the utility function and the production function that are consistent with balanced growth properties. Defend your choices.

(b) Specify first order autoregressive processes for z_t and g_t that have the desired unconditional means and variances.

(c) Formulate the planner's problem as a dynamic programming problem.

(d) It has been shown that for a calibrated version of the above model without government spending, the contemporaneous correlation between hours and productivity is close to one. Once stochastic government spending is added, the correlation becomes lower. Provide intuition for this finding.

(e) Formulate the problem that would be solved by a typical household and firm in a decentralized version of this economy. Define a *recursive competitive equilibrium* for this economy that includes a government that collects lump sum taxes and disposes of the proceeds.

(f) Is the competitive equilibrium you have defined equivalent to the allocation that would be chosen by a social planner? You do not need to provide a rigorous proof of this, but state your answer and provide some explanation. Suggest a modified version of this economy where the opposite result would hold.

(g) Now suppose that government expenditures are used to purchase consumption goods that are perfect substitutes for c_t in an agent's utility function. Specify the social planning problem for such an economy as a dynamic programming problem. Do you expect that the result described in part (d) would hold for this model? Explain.

Part II

1 Monetary Policy and Exchange Rates in a Growth Model

There are two consumption goods and two money stocks in the following perfect foresight, infinite horizon economy. Upper case letters are per-capita variables, and lower-case letters are household variables.

Preferences for the represenative household are given by

$$\sum \beta^t \{ \frac{\tilde{c}^{1-\eta}}{1-\eta} + v(l_t) \}$$

$$\tilde{c} = [\alpha c_a^\sigma + (1-\alpha) c_b^\sigma]^{\frac{1}{\sigma}}$$

The individual time constraint is

$$1 \geq l_t + h_{at} + h_{bt}$$

The economy's resource constraint is

$$\begin{aligned} A_a K_{at}^\theta H_{at}^{1-\theta} &\geq C_{at} \\ A_b K_{bt}^\theta H_{bt}^{1-\theta} &\geq C_{bt} + I_t \\ I_t &= K_{t+1} - (1-\delta) K_t \end{aligned}$$

Capital and labor can be costlessly reallocated across sectors.

There are two cash-in-advance constraints, one for good c_a and one for good c_b. The household CIA constraints are:

$$\begin{aligned} m_a &\geq p_a c_a \\ m_b &\geq p_b c_b \end{aligned}$$

Monetary policy for m_a and m_b is exogenous and given by

$$\begin{aligned} M_{at+1} &= \gamma_{at} M_{at} \\ M_{bt+1} &= \gamma_{bt} M_{bt} \end{aligned}$$

Monetary injections or withdrawls are implemented via lump-sum transfers

The per-capita endowments of m_a, m_b, and k at date 0 are given.

(1) What restriction(s) do you need to place on the function v? What restriction do you need to place on the parameter σ, and what features of the model do α and σgovern? For preferences over the consumption aggregate $\tilde{c}$, what restriction do you need to place on η? How do you need to modify the problem for the case of $\eta \to 0$?

(2) Write this problem as a competitive equilibrium. Solve for the first-order conditions. Define the object q as the *exchange rate* between the two moneys, m_a and m_b. Discuss how model parameters and state variables determine this exchange rate. Describe how this exchange rate would change over time as a function of γ_a and γ_b.

(3) Suppose that there was labor-augmenting technical progress in the two production functions, in which technological growth occured at a constant rate. Describe how you would construct a stationary equilibrium for this economy. (You can write down the equations if you prefer, otherwise, just state in words what you would do).

(4) Consider a steady state of this economy. Under what set of monetary policies does the competitive equilibrium coincide with the social optimum?

(5) Suppose that $\gamma_{at} = \varpi > 1$ for all t, and consider a steady state. Under what monetary policies is this economy equivalent to the cash good - credit good model of Lucas and Stokey? (Recall in Lucas-Stokey that there are multiple consumption goods, some of which require cash for purchase, while other consumptions goods and investment dont require cash).

(6) Consider a steady state, in which there are positive and constant rates of money growth for both currencies. Under these conditions, show that this economy is equivalent to the Lucas -Stokey model in which there is a specific tax and transfer policy in the Lucas-Stokey economy.

Part III

Human Capital and the Duration of Unemployment:

One of the puzzling features of the labor market in this recession is that there is a large stock of unemployed workers who have been unemployed for a long time. We explore how this might happen in a Mortensen-Pissarides search model with depreciation of skills during unemployment.

Time is continuous and labelled $t \geq 0$. There is a measure one of agents in the labor force (who can work). These agents in the labor force can be of one of two types: high productivity and low productivity. High productivity agents produce output y_H per unit time that they are employed in a match. Low productivity agents produce output $y_L < y_H$ per unit time that they are employed in a match.

These agents can also have one of two employment statuses: they can be employed in a match or unemployed. We assume that unemployed workers consume b per unit time while unemployed and that $y_H > b$. We will consider cases in which $y_L > b$ and $y_L < b$.

At each date t, let l_{Ht} denote the measure of agents who are employed in matches and have high productivity, l_{Lt} the measure of agents who are employed in matches and have low productivity, u_{Ht}, the measure of agents who are unemployed and have high productivity, and u_{Lt} the measure of workers who are unemployed and have low productivity. At each moment in time, we have $l_{Ht} + l_{Lt} + u_{Ht} + u_{Lt} = 1$.

Agents gain skills when they are matched in a job and lose skills when they are unemployed. Agents who are employed in a match also lose their jobs (their matches are destroyed) at an exogenous rate independent of their skill level and agents who are unemployed become employed at an endogenous rate that does depend on their skill level in a manner to be described below.

Specifically, assume that in steady-state employed agents lose their jobs at rate $\lambda > 0$ per unit time and that high skilled unemployed agents enter into a new match at rate $\alpha_{uH} > 0$ per unit time while low skilled unemployed agents enter into a new match at rate $\alpha_{uL} \geq 0$ per unit time. Low productivity agents who are employed in a match become high productivity agents at rate η per unit time (representing learning on the job) and high productivity agents who are unemployed become low productivity agents at rate δ per unit time (representing loss of human capital in unemployment).

Part A: Take the transition rates $\lambda, \alpha_{uH}, \alpha_{uL}, \eta$, and δ as parameters. Compute the steady state levels of l_H, l_L, u_H and u_L.

Consider two cases in your solution. In the first case, $\alpha_{uL} > 0$. That is, low skilled unemployed agents find jobs. In the second case, $\alpha_{uL} = 0$. That is, low skilled unemployed agents do not find jobs.

Give some intuition based on the parameters y_H, y_L and b as to why we might have $\alpha_{uL} = 0$ in an equilibrium.

Which parameter controls the duration of unemployment for low skilled unemployed agents? Show that the level of unemployment of low skilled agents (u_L) is increasing as the duration of unemployment for low skilled agents increases.

End of Part A

To hire unemployed agents, firms post vacancies. Firms can identify low and high skilled unemployed agents and thus can post separate vacancies for each type of worker. Let v_{Ht} denote the measure of vacancies posted at date t for high skilled unemployed agents and v_{Lt} denote the measure of vacancies posted at date t for low skilled unemployed agents. The measure of matches between unemployed agents and vacancies for each type of agent at t is given by $m(u_{Jt}, v_{Jt})$ for $J \in \{H, L\}$ with the standard properties (non-negative, increasing and strictly concave in each argument, $m(0, v) = m(u, 0) = 0$, and $m(u.v)$ constant returns to scale). We assume that each unemployed agent of each type finds a match at t at rate $\alpha_{uJt} = m(u_{Jt}, v_{Jt})/u_{Jt}$ for $J \in \{H, L\}$ and each firm with a vacancy of type J finds a match at t at rate $\alpha_{eJt} = m(u_{Jt}, v_{Jt})/v_{Jt}$. From here forward, we will consider steady-states of this economy in which u_{Jt}, v_{Jt} and the corresponding rates α_{uJt} and α_{eJt} are constant over time.

Firms that have posted a vacancy must pay $k > 0$ per unit time to post that vacancy. The productivity of a match of type J is y_J as described above.

Let $W_H(w_H)$ denote the value to a high productivity worker of being employed at wage w_H and let U_H denote the value to that worker of being unemployed. Let $J_H(y_H - w_H)$ denote the value to a firm of having a high productivity worker with productivity y_H hired at wage w_H and V_H denote the value of an unfilled vacancy for a high productivity worker.

Let $W_L(w_L)$ denote the value to a low productivity worker of being employed at wage w_L and let U_L denote the value to that agent of being unemployed. Let $J_L(y_L - w_L)$ denote the value to a firm of having a low productivity worker with productivity y_L hired at wage w_L and V_L denote the value of an unfilled vacancy for a low productivity worker.

Assume that all agents and firms discount the future at rate $r > 0$. Assume that a low productivity agent in a match who switches to being high productivity renegotiates his wage as a high productivity worker when the switch occurs.

Part B: Write the Bellman Equations for the firm and the worker defining $W_J(w)$, $J_J(y_J - w)$, U_J, and V_J for $J \in \{H, L\}$. Take care to consider the fact that agents can change productivities.

Assume that there is free entry into creating vacancies, so that $V_H = V_L = 0$. Also assume that low and high productivity workers have the same bargaining power. Use this condition and your Bellman equations to argue that with $y_L < y_H$, then we must have $\alpha_{eL} > \alpha_{eH}$. Use the fact that the matching function has constant returns to scale to argue that $\alpha_{uL} < \alpha_{uH}$.