PhD. Macroeconomics Qualifying Exam *Fall 2009*

Instructions: You have four hours to complete the exam. The exam has three parts. Each part counts for 1/3 of the exam. Part 3 has two questions. To receive full credit, you must answer all questions. Please answer each question in a separate blue book.

Question 1

Consider a standard neoclassical growth model where a single good is produced according to a Cobb-Douglas production function from capital and labor: $y_t = z_t k_t^{\theta} h_t^{1-\theta}$. Here, z_t is a random variable revealed at the beginning of period t. Output can be consumed (c_t) or invested (x_t), where $k_{t+1} = (1-\delta)k_t + x_t$.

A. The following are possible specifications for preferences:

(i) $$E\sum_{t=0}^{\infty}\beta^t\{\log c_t - A\frac{h_t^{\sigma}}{\sigma}\},\ A>0,\ \sigma>1$$

(ii) $$E\sum_{t=0}^{\infty}\beta^t\frac{(c_t - Ah_t^{\gamma})^{1-\sigma}}{1-\sigma},\ \gamma>1, A>0.$$

What does it mean to say that preferences are consistent with balanced growth? In each case, is the utility function consistent with balanced growth? Prove your claims formally (don't just quote a theorem).

B. In the real business cycle literature, a calibrated version of the stochastic neoclassical growth model is used to study aggregate fluctuations. How is the stochastic process for z_t calibrated? What data and/or statistics computed from data are used to calibrate this process? Explain.

C. Assuming preference specification (i) in part A, write down the first order condition for hours worked using the equilibrium result that the marginal product of labor is equal to the wage. Log-linearize this equation to express the (approximate) relationship between the percentage deviation from steady state of consumption, the wage rate, and hours worked that must hold along an equilibrium path.

Assuming (counter factually) that consumption is equal to its steady state value along an equilibrium path, describe how the value of σ affects the standard deviation of hours worked relative to the standard deviation of the wage rate. What value of σ would make this ratio as large as possible?

D. Assuming your answer to part C is correct, a value of σ that would lead to empirically plausible fluctuations in hours relative to wages would imply a labor supply elasticity that is too high compared to ones estimated in the micro labor literature. Describe one way (several exist in the literature) that the basic model can be modified in order to reconcile large fluctuations in hours relative to wages at the aggregate level with low micro estimates of the labor supply elasticity.

Macro Comprehensive Exam Question 2
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In this question, we examine the relationship between the money supply and the price level.

Consider the following cash-in-advance economy. Assume that there is no government debt or spending and that all money is injected through transfers to the households. Time is discrete and dencted $t = 1, 2, 3, \ldots$.

Uncertainty in this economy is modeled as a three-state Markov chain with states $\{\omega_1, \omega_2, \omega_3\}$. The probability that the state at date $t+1$ is ω_j given that the state today is ω_i is given by p_{ij}, with $p_{11} = 0$, $p_{12} = \rho$, $p_{13} = (1-\rho)$, $p_{21} = p_{31} = p_{23} = p_{32} = 0$, and $p_{22} = p_{33} = 1$. We assume that the initial state at date 1, denoted s_1 is $s_1 = \omega_1$. For $t \geq 1$, we let $s^t = (s_1, s_2, \ldots, t)$ denote the history of events through date t and $\pi_t(s^t)$ the associated date 1 probability of that history.

Every period, there is a constant aggregate endowment of a non-storable consumption good denoted $y_t(s^t) = y$. There is a representative household with preferences over consumption of this good given by

$$\sum_{t=1}^{\infty} \beta^t \sum_{s^t} u(c_t(s^t))\pi_t(s^t)$$

with $\beta \in (0,1)$. The agent has initial cash holdings M_0^g and no initial bond holdings.

At each date t and history s^t, the representative household trades money and state contingent bonds. The bonds denoted $B_t(s^t, \omega_i)$ mature in one period and pay one dollar (one unit of money) in the event that $s_{t+1} = \omega_i$ and nothing otherwise. The representative household faces a sequence of nominal bond prices $\{q_t(s^t, \omega_i)\}_{t=0}^{\infty}$ related to the nominal interest rate by

$$\sum_{i=1}^{3} q_t(s^t; \omega_i) = \frac{1}{(1 + i_t(s^t))}$$

The agent faces a sequence of budget constraints in the asset market for $t \geq 2$

$$\sum_{i=1}^{3} q_t(s^t, \omega_i) B_t(s^t, \omega_i) + M_t(s^t) =$$

$$P_{t-1}(s^{t-1})(y - c_{t-1}(s^{t-1})) + M_{t-1}(s^{t-1}) + B_{t-1}(s^t) + \tau_t(s^t)$$

and, for $t = 1$

$$\sum_{i=1}^{3} q_1(s^1, \omega_i) B_1(s^1, \omega_i) + M_1(s^1) = M_0^g + \tau_1(s^1)$$

where $\{\tau_t(s^t)\}_{t=1}^{\infty}$ is the sequence of government tax rates. In addition, the agent faces a lower bound on his bond holdings of $\bar{B}$. The agent also faces a sequence of cash-in-advance constraints

$$P_t(s^t)c_t(s^t) \leq M_t(s^t)$$

for all $t \geq 1$. The government's budget constraint is given by

$$\tau_t(s^t) = M_t^g(s^t) - M_{t-1}^g(s^{t-1})$$

The goods market clearing condition is $c_t = y$, the money market clearing condition is $M_t(s^t) = M_t^g(s^t)$, and the bond market clearing condition is $B_t(s^t, \omega_i) = 0$.

An equilibrium is a collection of sequences of agents' decisions $\{c_t(s^t), M_t(s^t), B_t(s^t, \omega_i)\}_{t=1}^{\infty}$, prices $\{q_t(s^t, \omega_i), P_t(s^t)\}_{t=1}^{\infty}$, and policy $\{M_t^g(s^t), \tau_t(s^t)\}_{t=1}^{\infty}$ such that the agents' decisions maximize the utility of the representative agent subject to the agents' budget constraints, cash-in-advance constraints, and the lower bound on agents' bondholdings and such that goods, money, and bond markets clear.

Part A: Prove that $\sum_{i=1}^{3} q_t(s^t, \omega_i) \leq 1$ in any equilibrium. Is it necessarily the case that $q_t(s^t, \omega_i) \leq 1$ in any equilibrium? If not why not?

Part B: Assume that $M_t^g(s^t) = \bar{M}(\omega_i)$ if $s_t = \omega_i$ for all $t \geq 1$ and s^t. Solve for the sequences of equilibrium bond prices $\{q_t(s^t, \omega_i)\}_{t=1}^{\infty}$.

Part C: What are the solutions for $P_t(s^t)$ for $t \geq 2$?

Part D: Explain how to solve for $P_1(s^1)$. There are two relevant cases that you should discuss: one in which the cash-in-advance constraint binds in period 1 and one in which it does not. Write down the conditions on $\bar{M}(\omega_i)$, β, and ρ that determine whether or not it binds.

Question 3A

Very recently, it was stated by an economics professor that "Economic theory tells us that the only case in which government spending doesn't raise real output is if government purchases are perfect substitutes for private spending."

Evaluate whether this statement is true or false when government finances tax revenue by taxing labor and capital income.

Question 3B

Consider a discrete-time, infinite horizon risk-sharing problem between a risk-averse agent and a risk neutral planner. The agent faces a stochastic endowment sequence $\{y_t\}$, where y_t is iid over time, and drawn from the set $\{y_1, ..., y_N\}$ with iid probabilities $\{\pi_1, ..., \pi_N\}$. The agent's endowment is privately observation, and transfers from the principal to the agent (and vice versa) can only be made contingent on voluntary information disclosure by the agent. The agent ranks consumption plans according to $\mathbb{E}\left\{\sum_{t=0}^{\infty} \beta^t u(c_t)\right\}$, where $\beta \in (0, 1)$, and $u(\cdot)$ is increasing, concave, and twice differentiable, with $\lim_{c \to 0} u'(c) = \infty$. The planner ranks the same sequence according to $\mathbb{E}\left\{\sum_{t=0}^{\infty} R^{-t} (y_t - c_t)\right\}$.

Set up the planning problem associated with this risk-sharing problem, and provide a sufficient condition for its solution. What are the implications for optimal consumption smoothing with private information?