Macroeconomic Theory

Ph.D. Qualifying Examination
Fall 2005

Comprehensive Examination **UCLA Dept. of Economics**

- **You have 4 hours to complete the exam.**
- **There are three parts to the exam.**
- **Answer all parts. Each part has equal weight.**

ANSWER EACH PART IN A SEPARATE BLUE BOOK.

PART ONE: ANSWER IN BOOK 1
WEIGHT 1/3

Consider an economy with a continuum of infinitely lived individuals each of who work one of two work shifts or not at all. The shifts correspond to working only a straight time shift (h_1) or straight time plus overtime ($h_1 + h_2$). Let n_{1t} be the fraction of individuals that work only straight time and n_{2t} be the fraction that work straight time plus overtime. Each individual has preferences given by $E\sum_{t=0}^{\infty}\beta^t\left[\log c_t + A\log(1-h_t)\right]$, where $h_t \in \left\{0, h_1, h_1 + h_2\right\}$.

Output, which can be used for consumption or investment, is produced using a Cobb-Douglas technology that is a function of capital, labor and a technology shock. That is, $y_t = e^{z_t}K_t^{\theta}(H_{1t}^{1-\theta} + H_{2t}^{1-\theta})$, where K_t is the stock of capital, H_{1t} is total straight time hours worked and H_{2t} is total overtime hours worked. Output can be used as consumption or investment (i_t). Investment in period t become productive capital in period t+1, and the stock of capital depreciates at the rate δ. Finally, the technology shock, z_t, evolves over time according to a first order autoregression, $z_{t+1} = \rho z_t + \varepsilon_{t+1}$, where $\varepsilon \sim N(0, \sigma_\varepsilon^2)$.

A. Write down the social planner's problem for this economy as a dynamic program. The planner should give equal weight to the utility of all individuals in its objective function.

B. Derive a set of first order necessary conditions that characterize a solution to this problem. In particular, find equations that determine the following variables:

y_t (output), c_t, n_{1t}, n_{2t}, H_t, i_t and K_{t+1}. Here, H_t is total hours worked. Also, characterize the steady state for a nonstochastic version of this problem.

C. Suppose that you are given the following statistics computed from U. S. data: (1) average labor's share; (2) the average capital to output ratio (annual); (3) the average investment to output ratio; and (4) the average fraction of time that individuals spend working in the market sector. Suppose that a period is one quarter of a year. Your job is to calibrate the economy so that the steady state matches the U.S. averages in these four respects. Show how these facts can be used to find values for A, β, θ, and δ. That is, give a set of equations that can be solved to obtain values for these parameters given values for h_1 and h_2.

D. For the set of equations you obtained in part B, write two of these equations in terms of log deviations from steady state. Derive a linear approximation of this set of two equations.

E. Suppose you combined the equations obtained in part B so that the linearized Euler equation is a function of only k_t, k_{t+1}, k_{t+2}, z_t, and z_{t+1}. The resulting equation will be of the form $0 = E_t[k_{t+2} + a_1 k_{t+1} + a_2 k_t + a_3 z_{t+1} + a_4 z_t]$. Show how this Euler equation can be solved for the optimal decision rule, $k_{t+1} = b_1 z_t + b_2 k_t$. In particular, find the parameters b_1 and b_2 as functions of $a_1, \ldots, a_4$. Does your solution satisfy the transversality condition? Explain.

F. Define a *recursive competitive equilibrium* for this economy.

PART TWO: ANSWER IN BOOK 2
WEIGHT 1/3

Answer all parts

1. Consider the following model in which there exist a representative family that maximizes the utility function:

(1) $$U = \sum_{t=1}^{\infty} \beta^{t-1} \frac{C_t^{1-\rho}}{1-\rho},$$

where ρ is a positive parameter, β is the discount factor and C_t is consumption. Output is produced from the technology:

(2) $$Y_t = A\left(\frac{M_t}{P_t}\right)^{\alpha},$$

where α is a positive parameter between zero and one, A is a positive parameter, M_t is the nominal quantity of money held between periods t and $t+1$ and P_t is the price of commodities in terms of money. All output is consumed. The family holds zero bonds and $\bar{M}$ units of money at the beginning of period 1.

The money supply is increased each period according to the rule:

(3) $$M_{t+1} = \mu M_t, \qquad \mu > 1, \qquad M_0 = \bar{M}$$

and all new money is distributed to the representative family as a lump sum transfer denoted T_t measured in units of money. The family may choose to hold its wealth in the form of money, or government bonds, B_t, where B_t represents bonds held between periods t and $t+1$. Bonds are in zero net supply. The nominal interest rate on a bond that is held between periods t and $t+1$ is denoted i_t. Let the present value of a period s dollar in period t be denoted by Q_t^s.

A. What is meant by a present value price? The term Q_t^t is defined to be identically equal to 1. Write an expression that defines Q_t^s as a function of the sequence of interest rates $\{i_j\}_{j=t}^{s}$ for $s > t$.

B. Assume that the family owns a representative firm and that each period it sells output, buys consumption goods and accumulates money and bonds in a sequence of markets. Write down the budget constraint faced by the family/firm in each of these markets.

C. Write down a single constraint for the family which constrains its expenditures and consumption plans for periods 1 through T. [HINT: this expression should involve terms in $\frac{M_t}{P_t}$, C_t, T_t, i_t, Q_1^t and Y_t, for $t = 1, \ldots T$].

D. What is meant by a Ponzi scheme? Write down a constraint that prevents the family from running a Ponzi scheme.

E. Write down a single infinite horizon budget constraint for the family. [HINT: this expression should involve terms in $\frac{M_t}{P_t}$, C_t, T_t, i_t, Q_1^t and Y_t, for $t = 1, \ldots \infty$].

F. Using equation (2) and your answer to part (B) rewrite the the utility function in terms of real balances $\frac{M_t}{P_t}$ and real bonds $\frac{B_t}{P_t}$. [Assume that the period budget constraint holds with equality.] Find a first order condition for the choice of real balances when the family firm maximizes the utility function you have derived written in terms of money and bonds.

G. Find a difference equation in real balances that must hold in a competitive equilibrium. [Hint: Use the equilibrium conditions, $C_t = Y_t$ and eliminate P_t from the problem using the money supply rule].

H. Find an expression for the unique steady state equilibrium value of real balances as a function of the parameters of the problem. Does an increase in the money growth rate raise or lower real balances? Provide intuition as to why this occurs.

I. What is meant by indeterminacy of equilibrium? Explain in words what condition is necessary for this steady state equilibrium to be indeterminate.

PART THREE: ANSWER IN BOOK 3
WEIGHT 1/3

Part III

ANSWER **ONE** of **TWO**

EITHER

1. Consider a version of our stationarized cash-in-advance economy in which the monetary injection is received at the beginning of the period before the goods market opens. After the goods market closes in the period, then the asset market opens in the second half of the period. In this case the household's problem is given by

$$V\left(m,b,\tau\right)=\max u\left(c\right)+v\left(1-l\right)+\beta\int V\left(m'/\left(1+\tau\right),b'/\left(1+\tau\right),\tau'\right)h\left(\tau'\mid\tau\right)d\tau'$$

subject to

$$pc+m'+q\left(s\right)b'\leq m+b+pl+\tau,$$

where h denotes the pdf on next period's money injection. The firm's problem is given by

$$\max_{l}\ pl-wl$$

The market clearing conditions are given by

$$c=l,$$
$$m'=l+\tau,$$
$$b'=0.$$

A) Characterize the equilibrium of this economy. Try to explain how the current money injection will affect the real variables in the economy. Please try and explain what is the impact of beliefs about τ' on current real variables.

B) Consider a perfect foresight version of this model. What monetary policies can lead to efficient equilibrium outcomes? Be sure to state what your notion of an efficient outcome is and why.

OR

2. Consider a version of the Ramsey problem of optimal taxation in which the government only has access to capital taxes, but not to labor taxes. Assume moreover, that the first period taxes on capital, τ_{k_0} is given exogenously. Assume that the model is otherwise standard. In particular, assume that the economy has the following features:

Government's budget constraint:

$$g_t + (1 + r_t) b_t = b_{t+1} + \tau_{k,t} q_t k_t.$$

Representative Household's problem

$$\max E \sum_t \beta^t u(c_t, l_t)$$

subject to

$$w_t l_t + (1 - \tau_{k,t}) q_t k_t + (1 + r_t) b_t = c_t + k_{t+1} - (1 - \delta) k_t + b_{t+1}$$

$$k_{t+1} \geq 0 \text{ and } k_0, b_0, r_0 \text{ given.}$$

Representative Firm's Problem:

$$\max \left[f(k_t, l_t) - q_t k_t - w_t l_t \right]$$

Resource Constraint:

$$c_t + k_{t+1} - (1 - \delta) k_t + g_t = f(k_t, l_t)$$

(Note that for simplicity we have assumed that depreciation isn't deductible.)

A) Construct the set of constraints that any primal approach to the government's problem must satisfy. Be sure to show that any allocation $\{c_t, l_t, k_{t+1}\}$ that satisfies these constraints can be mapped into a policy for the government and associated competitive equilibrium.

B) Set up the Lagrangian that characterizes the solution to the Ramsey problem, and derive the associated first-order conditions. What can you say about taxes in the long-run if this economy converges to a steady state in which government expenditures was constant at g?