Wednesday, June 23 2010

UCLA
Department of Economics
Ph.D. Preliminary Exam
Industrial Organization Field Exam
(Spring 2010)

Instructions:

- You have 4 hours for the exam.
- Answer any 5 out 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any one question. DO NOT answer all questions.
- Use SEPARATE booklets to answer each question
- Calculators and other electronic devices are not allowed.

1. **Competitive equilibrium with product differentiation.**

   Note. For this problem you are only required to answer the to first questions. The last one is for extra credit.

   Consumers. Goods can be of two possible quality levels, $l < h$. Consumers are as in the standard vertical differentiation model: they are distributed uniformly according to their preference for quality x in $[a, b]$ and have utilities $xl - p_l$ and $xh - p_h$ for low and high quality goods, respectively. At given prices, consumers choose to consume either high, low quality or no consumption

   Firms. The market is competitive. There is free entry into each of these markets. Firms are of measure zero relative to the market. Technology for production is given by fixed costs $F_l < F_h$ and variable costs $c_l(q) \leq c_h(q)$ which are increasing and strictly convex (note that this implies standard U-shaped average cost curves.)

   (a) Define a competitive equilibrium.

   (b) Give general conditions on the cost functions, l, h and the range $[a, b]$ so that there is entry in both levels of quality. Derive conditions that determine the mass of firms in each segment.

   (c) (extra credit) Suppose now that the fixed costs and variable costs are identical for both products, but that all entrants start low quality. Consider a continuous time model where it costs a firm $i(\lambda)$ to generate an arrival rate λ for upgrading its product. Once the upgrade arrives, the firm can only produce the high quality good. Make the necessary assumptions on $i(\cdot)$ so that the investment problem of the firm is well defined.

      i. Write down the dynamic programming problem faced by a firm (*Hint: it is a non-stationary problem so index value functions and profits by t*)

      ii. Conjecture that along the equilibrium path the value of being a high quality firm decreases monotonically over time. Show that the price of a low quality firm will rise over time. Is there exit along the path?

      iii. Suppose that the equilibrium converges to a situation where there are no low quality firms in the limit. How would you calculate this limiting number of firms?

2. **Jovanovic style model.** A competitive industry produces a homogenous good with aggregate demand $p = D(Q)$. The sequence of decisions is as follows. In the first period of entry, entrants pay a cost of entry c_e and draw $\theta \geq 0$ distributed according to an initial distribution $G(d\theta)$. After observing θ, they decide to stay or leave the industry and if stay, pay fixed cost ϕ and choose output q. The realized cost of a firm is $xc(q)$, where x is drawn from an exponential with cdf $F(x|\theta) = 1 - \exp(-\theta x)$ and c is strictly convex. In this first period of their lives, firms make their output choice prior to observing x. But the draw x from the first period remains the same forever. In the following periods, firms first decide whether to exit or stay, if stay pay fixed costs and produce (now knowing their cost parameter x.) Suppose firms also face an exogenous death shock that occurs with probabiity $(1 - \beta)$ and do not discount the future except for this.

   (a) Conjecture a constant price in the stationary equilibrium with entry and exit. Write down the value functions for entrants and for firms of age greater than one.

   (b) Let θ^* and x^* denote the corresponding exit points. Define a stationary equilibrium including the law of motion for the distribution of firm's states. (*Hint. Distinguish in the distribution entrants from older firms.)* Is the equilibrium unique?

   (c) Show that $E(x|\theta^*) = \frac{1}{\theta^*} > x^*$ and give an intuitive explanation for this difference.

   (d) Suppose now that β is a discount factor but there is no exogenous death and assum the supply function of a firm $q(x, p)$ is concave in x. Starting from no firms in the industry, show that the equilibrium path involves constant price and that the rate of entry and exit converges to zero.

3. Suppose that the government is considering imposing a 10% sales tax on a differentiated product industry. They have hired you to predict the effect of this tax on equilibrium prices and sales. Your are given historical data on the market over the past T time periods. Specifically, you have data on

   (a) s_{jt} - Market shares of the J products in each time period t (and s_{0t}, i.e. the outside good market share in each t)

   (b) X_{jt} - Product characteristics of the J products in each time period t.

   (c) p_{jt} - Prices of the J products in each time period t.

   (d) z_{jt} - a marginal cost shifter for each product j in each time period t.

   You are willing to assume that

   (a) Each product is produced by a different firm.

   (b) The firms engage in a static Nash-Bertrand pricing game in each time period.

   (c) Marginal costs are constant

   (d) Demand can be modelled using a standard logit model.

   (e) Unobserved demand shocks (or unobserved product characteristics) ξ_{jt} are mean independent of X_{jt} and z_{jt}.

   Carefully describe how you would estimate the potential effects of this tax on prices and market shares. To be a little more specific, describe how you could estimate what prices and market shares in period T *would have been* had the tax been enacted in period T.

4. Consider a 2 player binary choice game often used to model industry entry. Let $a_i \in \{0, 1\}$ denote the action of player i and a_{-i} denote the action of i's opponent. Suppose payoffs are given by:

$$U_i(a_i, a_{-i}) = \pi(a_i, a_{-i}) + \epsilon_i$$

where $\epsilon_i \sim F(\cdot; \theta)$ are i.i.d. Assume $\pi(0, a_{-i}) = 0$

(a) Suppose, first, that this is a game of complete information. Show graphically (ϵ_1 on the x-axis and ϵ_2 on the y-axis) the regions corresponding to monopoly, duopoly, and no entry. If there are regions of multiple equilibria indicate them on the graph.

(b) Suppose that we observe $\{a_1, a_2\}$ for a large number of markets. Consider the likelihood function, $L(a_1, a_2|\pi(1, 0), \pi(2, 0))$, what problems do multiple equilibria cause in defining the likelihood?

(c) Assume F is uniform on $[0, 1]$, write down a well defined likelihood function. You may redefine the dependent variable or write down inequality conditions.

Now suppose that the ϵ's are private information draws from a type 1 extreme value distribution (now a vector rather than a scaler). Suppose also that we observe a vector of firm level profit shifters $S = (S_1, S_2)$ so that payoffs are given by:

$$U_i(a_i, a_{-i}) = \pi(a_i, a_{-i}, S) + \epsilon_i(a_i)$$

(d) Show the model is not identified in the general case.

(e) Make whatever additional assumptions you want to make to ensure identification of the model and show the model is identified. Discuss a real world situation in which your assumptions are plausible.

5. Consider the first price sealed bid auction model with *independent private values*. There are N bidders with private values drawn from $v_i \sim F_i(v)$. Bidders are potentially risk averse and have utility functions $U(v) = v^\theta$. That is, bidders draw valuations from possibly different distributions but have common risk preferences. Note that $\theta = 1$ corresponds to risk neutrality. Assume throughout that we observe all bids and the identity of the bidders, there is no reserve price, and the number of bidders is fixed at N.

   (a) Suppose $\theta = 1$. Can we identify the distributions F_i? Show why or why not. (Hint: Start with the model expressed in terms of G_i, the cdf of equilibrium bids)

   (b) If the model is identified, describe a strategy for nonparametrically estimating the F_i's. If the model is not identified describe a strategy for the symmetric case and discuss where estimation goes bad in the asymmetric case.

   (c) Suppose $F_i = F$ but $\theta < 1$. Show the model, F, θ, is not identified. What assumptions do you need to identify the model?

   (d) Discuss strategies for identifying the general case, $\{F_i\}_{i=1,\ldots,N}, \theta$, relaxing whatever assumptions about the data you need to. Discuss the ways in which this situation is different from the the symmetric case in c..

6. Please, after you!

In a symmetric duopoly the NE payoff to each firm yields a profit rate of 1 to each firm. The interest rate is *r*. A new profit opportunity arises that will yield the firm that moves out of the original market a profit rate of 2. The other firm (that remains in the original market) will then have an even greater profit rate of $1+\theta$ where $\theta > 1$.

(a) Suppose firm 1 plans to move at time *t* (unless firm 2 has already moved) and firm 2 plans to move at time *s*. Confirm that if $s < t$ the present value of the profit stream for firm 1 is

$$A(s) = \frac{1}{r}(1-e^{-rs}) + \frac{1+\theta}{r}e^{-rs}.$$

Moreover if $s > t$ the present value of the profit stream for firm 1 is

$$B(t) = \frac{1}{r}(1-e^{-rt}) + \frac{2}{r}e^{-rt}.$$

(b) Explain why there can be no NE in pure strategies.

Let $G(\tau) \in \square^1$ be the symmetric NE probability that a firm moves by time τ so that firm 1's expected payoff is

$$\bar{U}_1(t) = \int_0^t A(s)G'(s)ds + (1-G(t)B(t).$$

(c) Show that the equilibrium probability distribution must have a "hazard rate" $h(t) = G'(t)/(1-G(t)$ that is time independent and so depends only on the parameters of the problem.

(d) What is the equilibrium expected payoff of each firm?

(e) How does the equilibrium mixed strategy change as θ and *r* change? Provide some intuition.

(f) What do think the equilibrium will be if there are three firms in the market. The one that moves increases its profit rate from 1 to 2 while the others become a duopoly and have a profit rate of 3 so again it is better to stay. (No formal analysis is expected.)