UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(Fall 2019)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

1. Substitutes and Complements

(a) Let $h(p, w)$ be a consumer's Hicksian demand function. Explain why h is homogeneous of degree 0 in p (i.e. $h(\alpha p, u) = h(p, u)$ for any $\alpha > 0$), and also why that implies $\sum_k \frac{\partial h_j}{\partial p_k} p_k = 0$ for any good j.

(b) Explain why $\frac{\partial h_k(p,u)}{\partial p_j} = \frac{\partial h_j(p,u)}{\partial p_k}$ must hold for any good k and good j (**hint:** use Shepard's lemma $\frac{\partial e(p,u)}{\partial p_k} = h_k(p, u)$).

(c) Good k and good j are substitutes when $\frac{\partial h_k}{\partial p_j} = \frac{\partial h_j}{\partial p_k} > 0$ and complements when $\frac{\partial h_k}{\partial p_j} = \frac{\partial h_j}{\partial p_k} < 0$. Explain why it is impossible for some good j to be a complement for all the other goods.

(d) Find an explicit example of utility function for which every good becomes a substitute for every other good.

2. A Simple Optimal Portfolio Problem: Suppose that an expected utility-maximizing investor with Bernoulli utility function $u(z) = -e^{-\alpha z}, \alpha > 0$ plans to invest $W > 0$ to either a safe asset with return 1 or a risky asset with random return $\widetilde{r}$ with mean $r = E[\widetilde{r}]$. This investor's investment problem can be expressed as follows:

$$\max_{A \in [0,W]} E[-e^{-\alpha(W-A+A\widetilde{r})}].$$

where A is the amount of wealth that is invested to the risky asset. Let $A^*(W)$ be the optimal solution for this problem. Answer the following questions.

(a) Show that this investor buys a risky asset (i.e. $A^*(W) > 0$) as long as the expected return r of the risky asset exceeds 1.

(b) Suppose that this problem has an interior solution $A^*(W) \in (0, W)$ given some W. What would happen to the optimal investment if the investor has twice more wealth to spend? Compare $A^*(2W)$ and $A^*(W)$.

(c) Suppose only for this question that $\widetilde{r}$ is normally distributed $\widetilde{r} \sim N(r, \sigma^2)$, where σ^2 is the variance. Derive the optimal $A^*(W)$, assuming an interior solution (**hint:** For any normal random variable $X \sim N(\mu, \sigma^2)$, $E[e^{bX}] = e^{b\mu + \frac{1}{2}b^2\sigma^2}$).

(d) Suppose instead that this investor's Bernoulli utility function is $u(z) = \log(z)$. Discuss how the optimal $A^*(W)$ would vary with the size of W in this case (**hint:** Express A as aW and try to find the optimal fraction of investment a^*).

3. Compromising: Two players $i = A, B$ split a pie of size one in a war of attrition. Formally, they choose concession times $T_A, T_B \geq 0$ and the player i with the larger time wins share $x_i \in [0, 1]$ of the pie, while the other player receives share $1 - x_i$; both players have a marginal delay cost of k and so incur total waiting costs of $k \min\{T_A, T_B\}$. We assume that winning is worthwhile, in that $x_i > 1 - x_{-i}$, or equivalently $x_i + x_{-i} > 1$. Assume for now that the shares x_i are exogenous.

(a) What are the pure strategy equilibria of this game?

(b) Solve for the unique mixed-strategy equilibrium where no player concedes with positive probability immediately, i.e. at $T_i = 0$.[1] What are the expected payoffs in this equilibrium?

Assume now that players i simultaneously choose their desired shares $x_i \in [x^*, 1]$ before playing the above war of attrition, for some parameter $x^* > 0.5$. Assume that they play the mixed-strategy equilibrium in part (b) in any subgame following any (x_A, x_B), and focus on the "first stage game" of choosing x_i.

(c) What shares x_i do they choose in equilibrium? What is the Pareto-optimal equilibrium?

[1] You can assume that the equilibrium distribution of T_i has no other atoms either, and no gaps either.

4 Conservatism: A manager privately observes a signal about a risky project $s \in [0, 1]$ and decides whether to implement it. If implemented, the "market" observes the outcome of the project: success or failure.

The manager can be competent or incompetent; nobody knows this type, not even the manager herself(!), but all believe she is competent with probability p. If the manager is competent, the signal s is informative and the project succeeds with probability s; if the manager is incompetent, the signal s is uninformative and the project succeeds with probability $1/2$.

As in Spence's signaling model, the "market" observes the manager's choice and (if the manager chose to implement the project) its outcome, but not the signal $s \in [0, 1]$; rather, the market believes $s \sim U[0, 1]$, uncorrelated with the type of the manager. Given this information, and the market's belief about the manager's strategy, the market updates its reputation, i.e. the assessment about the manager's competence, from p to p'.

The manager maximizes her expected reputation.

(a) From the manager's perspective, if she faces signal s and implements the project, what is the probability it will succeed?[2]

Consider from now on the strategy whereby the manager implements the project if and only if $s \geq \frac{1}{2}$.

(b) From the market's perspective, if the manager implements the project, what is the probability it will succeed?

(c) Calculate the market's posterior beliefs p' after observing (i) success, (ii) failure, (iii) non-implementation.

(d) Calculate the expected reputation from implementing given signal $s = 1/2$. Would the manager implement the project when she observes $s = 1/2$? Why is this surprising?

[2] Recall that the manager does not know whether or not she is competent, and believes that she is with probability p.

5. Incumbents vs. Entrants: A seller wishes to sell a single item, and chooses a mechanism to maximize her profit. The seller faces two potential buyers. Buyer 1 ("the incumbent") has known value $\theta_1 = 1$, while buyer 2 ("the entrant") has unknown value $\theta_2 \sim F[0, 2]$. The seller has value 0. If agent i obtains the good with probability P_i and pays transfer t_i, his utility is

$$u_i = P_i\theta_i - t_i$$

while the seller obtains profit

$$\pi = t_1 + t_2$$

(a) Describe the direct revelation mechanism (P_i, t_i).

(b) What is buyer 2's utility in an incentive compatible mechanism?

(c) What is the seller's profit?

(d) Assume

$$MR(\theta_2) = \theta_2 - \frac{1 - F(\theta_2)}{f(\theta_2)}$$

is increasing in θ_2. What's the seller's optimal mechanism? If $\theta_2 = 1$, which buyer gets the good?

6. Sequential Contests: A firm organizes a contest for two agents. If agent i exerts effort a_i then she wins with probability

$$p_i = \frac{a_i}{a_i + a_j}$$

The prize for winning is $W_1 \geq 0$ while the prize for losing is $W_0 \geq 0$. The cost of effort is $c(a_i) = a_i^2/2$. Agents are risk neutral, with utility

$$u_i = p_i W_1 + (1 - p_i) W_0 - c(a_i)$$

(a) Fix the prizes (W_1, W_0) such that $W_1 \geq W_0 \geq 0$. Characterize the symmetric NE efforts, a. What is the agents' expected utility, V, in this symmetric NE?

(b) Suppose the firm's profits equal efforts minus payments,

$$\pi = 2a - W_1 - W_0$$

Argue that the firm's profit maximizing prizes are $(W_1, W_0) = (1/4, 0)$. In this optimal contest, what is the agents' effort and the firm's optimal profit?

Now suppose the firm organizes a two-stage knockout contest for four players. There are two semi-finals. In the first semi-final, agents 1 and 2 compete. In the second semi-final, agents 3 and 4 compete. The winner of each goes into the final. The prizes are as follows: the winner of the final gets $W_2 \geq 0$; the loser of the final gets $W_1 \geq 0$; the losers of the semi-finals get $W_0 \geq 0$.

(c) Fix the prizes (W_2, W_1, W_0) such that $W_2 \geq W_1 \geq W_0 \geq 0$. Characterize the symmetric Nash equilibrium efforts in the semi-final, a_S, and the final, a_F.

(d) Suppose the firm's profits equal efforts minus payments,

$$\pi = 2a_F + 4a_S - W_2 - W_1 - 2W_0$$

In the two-stage contest, there are three games rather than one. Suppose we set $(W_2, W_1, W_0) = (3/4, 0, 0)$, so as to keep the prize budget three times that in (b). Show that equilibrium effort is higher in both stages of the two-stage contest than in the one-stage contest, and hence profits are more than three times higher in the two-stage contest. Intuitively, why is the two-stage contest better than the one-stage contest?