UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2019)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

1. Random Choice: Anna always makes her choice in the following way. From any given finite set of objects $B \subset X$ (where X is the finite set of all possible objects) she first picks two items completely randomly, then compares them and chooses the better one (assume that she has a strict preference $\succ$ over X). Note that this choice procedure generates a choice distribution over B for each choice problem B in contrast to deterministic choice rules we learned in class, that associate an element/a set of elements with each B. Answer the following questions.

(a) Suppose that Anna is choosing one object from $\{a, b, c, d\}$ and that they are ordered as $b \succ c \succ d \succ a$ according to her strict preference. What is the probability of Anna choosing a, b, c, and d from this set?

(b) In the above example, a more preferred alternative would be chosen with a higher probability. Explain why this is in general true for any choice problem $B' \subset X$.

(c) Suppose that Anna adopts a slightly more sophisticated procedure: first pick three options randomly, then choose the best one. Derive the choice distribution that is generated by this choice procedure for the choice problem in (a).

(d) Suppose that the monetary value of a, b, c and d is \$1, \$20, \$10 and \$5 respectively for Anna. So we can turn any distribution (p, q, r, s) on $\{a, b, c, d\}$ into the corresponding distribution (p, s, r, q) on $\{1, 5, 10, 20\}$ that generates the same expected utility for Anna. Show that the distribution in (c) over $\{1, 5, 10, 20\}$ first order stochastically dominates the distribution in (a) by using the definition of FOSD or one of its equivalent conditions.

2. Walrasian Equilibrium: Consider an exchange economy with two goods A,B and two consumers $1, 2$. Suppose that consumer i's preference is represented by the following utility function: $u(x_i) = 0.5\log(x_{A,i} - \gamma_{A,i}) + 0.5\log(y_{B,i} - \gamma_{B,i})$ for $i = 1, 2$, where $\gamma_{A,i}, \gamma_{B,i} > 0$ are some strictly positive numbers that are fixed throughout this question. Assume that i's initial endowment $e_i \in \mathbb{R}^2_+$ is always strictly above γ_i (i.e. $e_i = (e_{A,i}, e_{B,i}) \gg (\gamma_{A,i}, \gamma_{B,i})$) for $i = 1, 2$. Answer the following questions.

(a) What is consumer i's (Walrasian) demand function $x_i(p, p \cdot e_i)$?

(b) Verify that the aggregate demand $x_1(p, p \cdot e_1) + x_2(p, p \cdot e_2)$ only depends on price p and the total resource in the economy $r = e_1 + e_2$ (and $\gamma_i, i = 1, 2$). That is, the aggregate demand would not change even if the resources are redistributed.

(c) Suppose that consumers' endowments are given by $e_1 = (e_{A,1}, e_{B,1}) = (2, 4)$ and $e_2 = (e_{A,2}, e_{B,2}) = (4, 2)$. Find a Walrasian equilibrium in this exchange economy.

(d) Explain why there is no other Walrasian equilibrium for (c) (after price normalization). Also show that the equilibrium price ratio $\frac{p_A^*}{p_B^*}$ would be the same for any e_1', e_2' such that $e_1' + e_2' = e_1 + e_2 = (6, 6)$.

3. Too good to fire: A firm employs a worker for potentially infinitely many periods $t = 0, 1, 2, 3....$ In every period t, first the worker chooses effort $e_t \in \{0, 1\}$, and then the firm chooses whether to fire the worker, and thereby ending the game, or to retain him, in which case play proceeds to period $t + 1$. We assume complete and perfect information. Per-period payoffs are $w - e_t$ for the worker and $r(e_t) - w$ for the firm, where $w > 1$ is an exogenous wage level, and $r(e)$ an exogenous revenue function that satisfies $r(0) \leq w < r(1)$. Overall payoffs are additive across periods, discounted at a common rate $\delta < 1$: So if the firm fires the worker in period $T \in \mathbb{N} \cup \{\infty\}$, payoffs are $\sum_{t=0}^{T} \delta^t (w - e_t)$ for the worker, and $\sum_{t=0}^{T} \delta^t (r(e_t) - w)$ for the firm; i.e. payoffs are still collected in the firing period T. The solution concept is SPE, which we'll simply call "equilibrium".

(a) Find an equilibrium where the worker never exerts effort, $e_t = 0$.

(b) For what discount factors δ is there another equilibrium, in which the worker always exerts effort, $e_t = 1$, on the equilibrium path? (Make sure to specify the worker's off-path actions in this equilibrium and argue why the firm's strategy in your equilibrium is optimal for the firm)

Assume from now on that the employee is so awesome that it is worthwhile to employ him even when he shirks, i.e. $r(0) > w$.

(c) Argue that the strategy profiles you constructed in parts (a) and (b) are no longer equilibria.

(d) Show that there is a unique equilibrium and describe this equilibrium.

(e) Now assume that before playing the game, the firm can raise the wage once-and-for-all, i.e. choose a wage level $w' \geq w$, and then the above game is played with wage w' instead of w. Also assume $r(1) - r(0) > r(0) - w$, and that $\delta \geq \delta^*$, where δ^* is the lower bound on the discount factor from part (b). Show that there is an equilibrium where the firm strictly prefers to raise the wage.

4 Political Correctness: The president is seeking advice from an economist on a policy. The timing of the game is as follows. First, the economist privately observes his type $\theta \in \Theta = \{B, F\}$ (benevolent or fanatic), and the state of the world $s \in S = \{0, 1\}$. Second, the economist sends a cheap-talk message $m \in M = \{0, 1\}$ to the president. Third, the president chooses the policy $a \in A = [0, 1]$. The president does not know θ and s, and assigns independent probabilities $p = \Pr(s = 1)$ and $q = \Pr(\theta = B)$ to them. Payoffs are as follows: The president and the benevolent economist want to match the action to the state, $u_P(a, s) = u_{E,B}(a, s) = -(a-s)^2$, whereas the fanatic economist always prefers the high action, $u_{E,F}(a, s) = -(a-1)^2$. We will solve for (weak) PBE of this game, but simply say "equilibrium".[1]

(a) Describe the strategy sets for the president and the economist.

(b) Assume that the economist babbles by randomizing 50-50 over the messages $m \in \{0, 1\}$ irrespective of his type or the state. What is the president's updated belief $p'(m)$ over $S = \{0, 1\}$ after each message m? Show that the president optimally chooses $a(m) = p$ after any message m. Can this behavior be part of an equilibrium?

(c) Assume the benevolent economist is truthful, i.e. sends messages $m = s$, while the fanatic economist sends message $m = 1$ irrespective of the state. What are the updated beliefs $p'(m)$ over $S = \{0, 1\}$ after messages $m = \{0, 1\}$? Solve for the president's optimal responses $a(0)$ and $a(1)$. Is this behavior & beliefs an equilibrium? Show that $a(1)$ is increasing in q. Interpret this finding.

(d) Calculate the economist's reputation, that is, the president's posterior expected belief about the economist's type $q'(m) = \Pr(\theta = B|m)$, conditional on message m (given the strategies in part c).

[1] Note that unlike in a signaling game, the messages m are "cheap talk" in that they do not directly enter the players' preferences, but matter only via the induced actions.

5. Akerlof with Two Types: A seller wishes to sell his car. The car's type θ is privately known by the seller. Two potential buyers $i \in \{1,2\}$ simultaneously offer prices p_i. The seller can choose either offer, or keep the car.

Assume $\theta \in \{5, 10\}$ with equal probability. Each buyer values the car at θ. A seller of type θ values the car at $r(\theta)$.

For each of the three cases below: (i) Describe which types sell, $\Theta(p)$, as a function of the highest price $p = \max\{p_1, p_2\}$, and (ii) Describe the PBE of the pricing game (and explain your reasoning). The three cases:

(a) $r(5) = 3$ and $r(10) = 6$.

(b) $r(5) = 4$ and $r(10) = 8$.

(c) $r(5) = 8$ and $r(10) = 4$.

6. Contracting with Externalities: Two firms ("agents") would like to invest in a corrupt country. The Government ("principal") has the bargaining power, and is willing to let the firms invest in exchange for bribes. Formally, a contract $\langle x_i, t_i \rangle$ with agent i describes the investment level $x_i \geq 0$ and bribe $t_i \geq 0$, both of which are contractible.

The principal makes profits $\pi = t_1 + t_2$. If agent i accepts the contract, he makes utility $u_i = v_i - t_i$, where $v_i = x_i - \frac{1}{2}x_i^2 + \alpha x_j$. If agent i rejects the contract, he makes utility $u_i = v_i = \alpha x_j$. Note that $\alpha > 0$ means the firms have positive externalities on each other, while $\alpha < 0$ means the firms have negative externalities on each other. We assume that $\alpha > -1$.

(a) What is the Pareto efficient investment level?

(b) Suppose the principal offers a "multilateral" contract $\langle x_i, t_i \rangle$ to each agent simultaneously. Formally: (i) the principal offers each agents a contract $\langle x_i, t_i \rangle$. (ii) Both choose to accept or reject. (iii) If *either* agent rejects then both contracts are cancelled. What are the principal's optimal choices of x_1 and x_2?[2]

(c) Suppose the principal offers "bilateral" contract $\langle x_i, t_i \rangle$ to each agent simultaneously. Formally: (i) the principal offers each agent a contract $\langle x_i, t_i \rangle$. (ii) Both choose to accept or reject. (iii) If an agent rejects then the other agent's contract is unaffected. What are the principal's optimal choices of x_1 and x_2?

(d) How does the level of investment under multilateral and bilateral contracts depend on α? Provide an intuition.

(e) How does the principal's profit under multilateral and bilateral contracts depend on α? Provide an intuition.

[2] We wish to characterize the principal's best equilibrium and are thus doing "partial implementation". There is trivially an equilibrium where both agents reject; we will ignore this.