UCLA Department of Economics **Spring 2019**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. All three parts will receive equal weight in your grade. **Answer each part in a separate bluebook.**

Part 1

Consider a stochastic growth economy with one representative agent and three production sectors—two market sectors and one nonmarket (home production) sector. Each of two market sectors employs labor and two types of capital: equipment and structures. Denote these by H, K_E and K_S. Sector 1, the consumption good sector, produces a market consumption good, C_M, and structures using the technology

(*) $$C_{M,t} + K_{S,t+1} = Y_{1,t} = z_t K_{E,1,t}^{\theta_1} K_{S,1,t}^{\theta_2} H_{1,t}^{1-\theta_1-\theta_2} + (1-\delta_S)K_{S,t}.$$

Here, z_t is technology shock where $\log z_{t+1} = \rho_z \log z_t + \varepsilon_{1,t+1}$, $\varepsilon_1 \sim N(0,\sigma_1^2)$.

The second sector, sector 2, uses the same inputs to produce equipment and consumer durables, D_t. The technology and resource constraint for this sector is

(**) $$D_{t+1} + K_{E,t+1} = Y_{2,t} = q_t z_t K_{E,2,t}^{\theta_1} K_{S,2,t}^{\theta_2} H_{2,t}^{1-\theta_1-\theta_2} + (1-\delta_E)K_{E,t} + (1-\delta_D)D_t.$$

In this sector, $q_t z_t$ is the technology shock and $\log q_{t+1} = \rho_q \log q_t + \varepsilon_{2,t+1}$, $\varepsilon_2 \sim N(0,\sigma_2^2)$. Notice that the Cobb-Douglas production function combining capital and labor are identical in the two sectors.

In the home, households combine consumer durables and labor (L) to produce a nonmarket consumption good, C_N. In particular, $C_{N,t} = F(D_t, L_t)$, where F is a constant returns to scale production function. Households have one unit of time to divide between market work, home work, and leisure. Preferences are given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha \log C_{M,t} + (1-\alpha)\log C_{N,t} + A\log(1-H_t-L_t) \right].$$

A. Carefully formulate the dynamic program that would be solved by a social planner in this economy. Be sure to be clear about the state variables and choice variables.

B. Derive expressions that determine how the planner allocates a given amount of capital and labor across the two market sectors. Prove that the same fraction of each input is allocated to a given sector in period *t*. That is, show that $H_{1,t} = \phi_t H_t$, $K_{E,1,t} = \phi_t K_{E,t}$ and $K_{S,1,t} = \phi_t K_{S,t}$. Obviously the remainder, a fraction $(1-\phi_t)$, is allocated to sector 2.

C. Show that the result obtained in part B can be used to aggregate the resource constraints (*) and (**) into one resource constraint (derive it). Repeat part A given this result.

D. Define a *recursive competitive equilibrium* for this economy. Be sure there are markets for both market goods. What is the relative price of the output of sector 2? Explain.

E. Suppose an empirical fact about the world is that as durable goods become cheaper, households spend less time in nonmarket activities (female labor supply increases, for example). If you wanted this model to be consistent with this fact, what would this mean for the functional form you would choose for the home production technology, *F*? (Note: This question is not so much asking for a functional form for *F*, although that would be fine, but is asking for list of properties that this function should possess.)

Part 2. (10pt in total)

In this problem you will study asset pricing in a stochastic growth model with overlapping generations. Time is discrete, $t \in \{1, 2, \dots\}$ and the horizon infinite. Every period, a stochastic event z is drawn from some finite set Z. This event will correspond to realizations of aggregate productivity. We denote a time-t history by $z^t = (z_1, z_2, \dots z_t) \in Z^t$. We assume that draws are IID over time with distribution $\pi(z_t)$.

Every period, there is a one-period lived representative competitive firm who rents capital and hires labor in order to produce output according to the Cobb-Douglas production function: $Y_t(z^t) = z_t K_t(z^t)^\alpha L_t(z^t)^{1-\alpha}$, where $K_t(z^t)$ and $L_t(z^t)$ denote capital and labor demand. Capital depreciates fully every period after it is used for production.

Every period, a measure one of two-period lived households is born. Each household is endowed with one unit of labor when young, and no labor when old. It supplies labor inelastically, chooses how much to consume, when young, $c_t^y(z^t)$, how much consumption goods to save in the form of capital when young, $k_{t+1}(z^t)$, how much consumption goods to save in the form of risk-free bonds when young, $b_t(z^t)$, and how much to consume when old, $c_{t+1}^o(z^{t+1})$, in order to maximize its inter-temporal utility:

$$\log\left(c_t^y(z^t)\right) + \beta \sum_{z_{t+1}\in Z} \pi(z_{t+1}) \log\left(c_{t+1}^o(z^t, z_{t+1})\right).$$

The sequential budget constraints of a young household born at time (t, z^t) are:

$$\begin{aligned}
&c_t^y(z^t) + k_{t+1}(z^t) + b_t(z^t) \leq W_t(z^t) \\
&c_{t+1}^o(z^{t+1}) \leq R_b(z^t) b_t(z^t) + R_{kt+1}(z^{t+1}) k_{t+1}(z^t).
\end{aligned}$$

where $W_t(z^t)$ is the wage rate at time (t, z^t), $R_{bt}(z^t)$ is the gross rate of return on risk-free bond, and $R_{kt+1}(z^{t+1})$ is the gross rate of return on the capital. Since capital is rented to firms and then depreciates fully, $R_{kt+1}(z^{t+1})$ is also the rental rate of capital. Finally, there is a measure one of initial old at time zero who have a strictly increasing utility over time 1 consumption, $c_1^o(z_1)$, and who are endowed the capital stock K_1. Risk-free bonds are in zero net supply.

Before proceeding to the questions, keep in mind that, since the aggregate capital stock in period $t+1$ is determined by the aggregate saving decision of young households at t, in equilibrium $K_{t+1}(z^{t+1})$ is known in period t. Hence, the only uncertainty faced by young agents born at t is over the realization of z_{t+1}.

1. Define the following problems: the problem of the representative firm; the problem of the initial old household, and the problem of a household born at time (t, z^t). (1pt)

2. Define an equilibrium. (1pt)

3. Derive the first-order conditions for problem of the representative firm operating at (t, z^t). (1pt)

4. Derive the first-order conditions for the problem of a household born at time (t, z^t). (1pt)

5. Using the first-order conditions for the problem of a household born at time (t, z^t), show that, in equilibrium, the saving rate is constant, that is: (1pt)

$$k_{t+1}(z^t) = \frac{\beta}{1+\beta} W_t(z^t) \text{ and } c_t^y(z^t) = \frac{1}{1+\beta} W_t(z^t).$$

   (if you get stuck with this question, just move on and use the result for what follows!)

6. Combine the first-order conditions for the problem of a representative firm, the result of question 5, and the budget constraint of the household to show that (1pt)

$$c_{t+1}^o(z^{t+1}) = z_{t+1} \alpha K_{t+1}(z^{t+1})^{-(1-\alpha)} \beta c_t^y(z^t).$$

   (if you get stuck with this question, just move on and use the result for what follows!)

7. Use the first-order conditions of a household to solve for the gross return of a risk free bond at time (t, z^t), $R_{bt}(z^t)$, as a function of $K_{t+1}(z^{t+1})$ and $\left(\sum_{z_{t+1} \in Z} \pi(z_{t+1}) z_{t+1}^{-1}\right)^{-1}$. (1pt)

8. Suppose there is an increase in uncertainty, modeled as a second-order stochastic dominance shift in the distribution of z_{t+1}. Holding $K_{t+1}(z^{t+1})$ constant, does the risk-free rate increase or decrease? Why? (1pt)

9. Solve for the realized excess return on capital, $R_{kt+1}(z^{t+1}) - R_{bt}(z^t)$ as a function of $K_{t+1}(z^{t+1})$, z_{t+1}, and $\left(\sum_{z_{t+1} \in Z} \pi(z_{t+1}) z_{t+1}^{-1}\right)^{-1}$. (1pt)

10. Suppose there is an increase in uncertainty, modeled as a second-order stochastic dominance shift in the distribution of z_{t+1}. Holding $K_{t+1}(z^{t+1})$ constant, does the expected excess return on capital increase or decrease? Why? (1pt)

Part 3 - Economic Growth with Multiple Capital Goods

Please read the entire question before writing your answers.

Write your answers carefully. Illegible and/or unclear answers will not receive credit.

Consider the following economy. Preferences for a representative consumer are given by:

$$\max \sum_{t=0}^{\infty} \beta^t \ln(c_t)$$

Final output is produced from two types of capital as follows:

$$y_t = Ak_{1t}^{\theta} k_{2t}^{1-\theta}$$

Final output is divided between consumption and investment:

$$y_t = c_t + i_{1t} + i_{2t}$$

The following laws of motion for the capital stocks are given as follows:

$$\begin{aligned}
k_{1t+1} &= (1-\delta_1)k_{1t} + i_{1t} \\
k_{2t+1} &= (1-\delta_2)k_{2t} + Bi_{2t} \\
\delta_1 &= \delta_2, B = 1
\end{aligned}$$

Note that the parameter B is a technology parameter. For this part of the test, you will assume that $B = 1$, and that $\delta_1 = \delta_2$. Later, we will change those assumptions.

A. Formulate this economy as a social planning problem and solve for the first order necessary conditions. (5 points)

B. Formulate this problem as a competitive equilibrium problem, assume that the numeraire is the consumption good, define the equilibrium, and show that the equilibrium is first best. Your equilibrium should include two firms, one of which produces the first capital good using final output, and the second which produces the second capital good using final output. (8 points)

C. Does a steady state exist for this economy? Or a steady state growth path? Explain your reasoning. (7 points)

D. Show that this economy is identical to an "AK" economy that has a single capital stock. (5 points)

Now, consider the following changes to this economy:

$$
\begin{aligned}
k_{1t+1} &= (1-\delta_1)k_{1t} + i_{1t} \\
k_{2t+1} &= (1-\delta_2)k_{2t} + Bi_{2t} \\
\delta_1 &\neq \delta_2, 0 < B < \infty, B \neq 1
\end{aligned}
$$

E. Using the results you derived previously, provide a formula for the relative price of k_2, and discuss the economic intuition behind this relative price. Explain how the value of B and the difference between δ_1 and δ_2 affect the incentives to accumulate the second capital good and how they affect the relative quantities of the two types of capital goods (7 points).

F. Does a steady state exist for this economy? Or a steady state growth path? Explain your reasoning, and include a discussion of how the value of B and the difference between δ_1 and δ_2 affect your results. (5 points)