UCLA Department of Economics **Spring 2018**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider a problem solved by a planner who maximizes the following:

$$E\sum_{t=0}^{\infty}\beta^t U(c_t, 1-h_t).$$

Here, c_t is consumption, h_t is hours worked, the function U is assumed to have all the usual properties, and $0 < \beta < 1$. Output is produced according to a constant returns to scale technology, $y_t = z_t F(k_t, h_t)$, where y_t is output and k_t is the stock of capital. The variable z_t is a technology shock that evolves through time according to a two-state symmetric Markov chain with an unconditional mean $\mu_z = 1$, an unconditional variance of σ_z^2, and an unconditional first order serial autocovariance $\text{cov}(z_{t+1}, z_t) = x_z$. The stock of capital is assumed to depreciate at the rate δ each period.

Output can be used for consumption, investment (i_t) or government purchases (g_t). Investment in period t becomes productive capital one period later, $k_{t+1} = (1-\delta)k_t + i_t$. The stochastic process for government spending is also a two-state Markov chain with an unconditional mean of $\mu_g = \overline{g}$, an unconditional variance of σ_g^2, and an unconditional first order autocovariance equal to x_g. Assume, initially, that government purchases do not directly affect preferences or the technology; they are simply thrown into the sea.

(a) Suggest functional forms for the utility function and the production function that are consistent with balanced growth properties. Defend your choices.

(b) Specify the parameters of the Markov chains for z_t and g_t in terms of the given first and second moments.

(c) Formulate the planner's problem as a dynamic programming problem.

(d) It has been shown that for a calibrated version of the above model without government spending, the contemporaneous correlation between hours and productivity is close to one. Once stochastic government spending is added, the correlation becomes lower. Provide intuition for this finding.

(e) Formulate the problem that would be solved by a typical household and firm in a decentralized version of this economy where government purchases are financed with lump sum taxes. Define a *recursive competitive equilibrium* for this economy.

(f) Is the competitive equilibrium you have defined equivalent to the allocation that would be chosen by a social planner? You do not need to provide a rigorous proof of this, but state your answer and provide explanation of how you would go about proving this.

(g) Now suppose that government expenditures are used to purchase consumption goods that are perfect substitutes for c_t in an agent's utility function. Specify the social planning problem for such an economy as a dynamic programming problem. Do you expect that the result described in part (d) would hold for this model? Explain.

Part 2. (10pt in total)

This part is based on a paper by Krishnamurthy and Vissing Jorgensen (JPE, 2010). You will study the pricing of government debt when it provides liquidity services to households.

The following elements of the setup are the same as in the class notes. Time is discrete and the horizon infinite. Every period, a stochastic event s is drawn from a finite set S. We denote a time-t history by $s^t \in \{s_0\} \times S^{t-1}$, and we let the probability distribution over histories be $\pi_{0t}(s^t)$. Every period, there are two types of assets. First, there is a complete set of Arrow securities in zero net supply. As in class, we will denote by $Q_{t+1}(s \mid s^t)$ the price of a one-step ahead Arrow security, purchased at time t after history s^t, that pays one unit of consumption good only if state s realizes at time $t+1$. Second, there is one-period risk-free debt issued by the government in supply $B_t(s^t)$. Each unit of government debt pays one unit of consumption good next period, regardless of the state. The price of one unit of government debt is denoted by $P_t^B(s^t)$. The payments made on the debt are financed by lump-sum taxes. There is a single representative household who receives, at each time t and after every history s^t, the endowment $y_t(s^t)$. Every period, the household chooses its consumption, $c_t(s^t)$, the amount of Arrow securities it wants to hold, $a_{t+1}(s^t, s)$ for each state $s \in S$, the amount of government debt it wants to hold, $b_t(s^t)$, and pays lump sum taxes $\tau_t(s^t)$.

The only difference between the present setup and the one in the class notes is the following assumption: we assume that the representative household derives utility over government debt. Namely, the intertemporal utility of the household is:

$$\sum_{t \geq 0} \sum_{s^t \in S^t} \beta^t \pi_{0t}(s^t) U\left[c_t(s^t), b_t(s^t)\right], \text{ where } U[c, b] \equiv u\left[c \times \phi\left(b/c\right)\right]$$

for some functions $u(x)$ and $\phi(x)$ that are both bounded, strictly increasing, strictly concave, and twice continuously differentiable. Finally, assume that at time zero, the household starts with no holding of Arrow securities or government debt, that is, $a_0(s_0) = b_{-1} = 0$.

1. (1pt) Define the household's sequential budget constraint and the household's problem (you do not need to explicitly state the no Ponzi game condition).

2. (1pt) Define the government sequential budget constraint (you do not need to explicitly state the no Ponzi game condition).

3. (1pt) Define an equilibrium.

4. (1pt) Take first-order conditions for the household's problem. To simplify formula, it is useful to introduce the notation $\theta_t(s^t) = b_t(s^t)/c_t(s^t)$ for the ratio of debt to consumption. *Do not forget that c also enters in the utility via the ratio b/c in the function $\phi(b/c)$.*

5. (1pt) Find a formula for $Q_{t+1}(s_{t+1} \,|\, s^t)$, in terms of the aggregate endowment, and of the ratio of aggregate debt to aggregate endowment, $\Theta_t(s^t) \equiv B_t(s^t)/y_t(s^t)$.

6. (1pt) Consider some hypothetical one-period risk-free asset that does not enter the utility function, and let its price be $P_t^F(s^t)$. Find a formula for $P_t^F(s^t)$ in terms of $Q_{t+1}(s_{t+1} \,|\, s^t)$.

7. (2pt) Find a formula for $P_t^B(s^t) - P_t^F(s^t)$, the difference between the price of one-period government debt, and of the one-period risk-free asset that does not enter in the utility function. What is the sign of $P_t^B(s^t) - P_t^F(s^t)$? How does $P_t^B(s^t) - P_t^F(s^t)$ depend on the ratio of aggregate debt to aggregate endowment, $\Theta_t(s^t)$? Why?

8. (1pt) In reality it is difficult to find truly risk-free assets beside U.S. government debt, so measuring $P_t^B(s^t) - P_t^F(s^t)$ is not an obvious task. In line with this observation, suppose that, instead of a truly risk-free asset that does not enter the utility function, one uses some one-period corporate bond. Write the payoff of the corporate bond as $1 - D_{t+1}(s^{t+1})$, where $D_{t+1}(s^{t+1})$ is the size of corporate default. Let $P_t^C(s^t)$ denote the price of this corporate bond. Find a formula for $P_t^B(s^t) - P_t^C(s^t)$. Is it larger or smaller than $P_t^B(s^t) - P_t^F(s^t)$? Suppose that an econometrician proxies for $P_t^F(s^t)$ using $P_t^C(s^t)$. Explain in words what are the biases that the econometrician should worry about.

9. (1pt) Assume that $u(x) = x^{1-\gamma}/(1-\gamma)$ and that the government keeps a constant aggregate debt to aggregate endowment ratio, that is, $\Theta_t(s^t) =$ constant. As in class, let the equity premium be the difference between the expected return of the stock market, and the return of one-period government debt. Holding everything else equal, would the equity premium be larger in this model than in the model we studied in class? According to this model, what could be a new explanation the equity premium puzzle (new relative to what we've done in class)?

Part C - Optimal Growth in a Monetary Economy

Each sub-question carries equal weight.

In the economy below, refer to lower-case letters as household choices, and upper-case letters are per-capita variables.

The representative household has the following objective:

$$\max \sum_{t=0}^{\infty} \beta^t \{\ln(c_t) - \phi h_t\}$$

Consumption must be purchased with pre-accumulated cash and a monetary transfer from the government:

$$m_{t-1} + T_t \geq P_t c_t$$

Monetary policy is given by:

$$M_t = M_{t-1} + T_t,$$

where T is a lump-sum monetary transfer to the household, which can be positive, negative, or zero, and which is implemented at the start of each period.

The production technology and resource constraint are given by:

$$Y_t = AK_t^{\theta}(X_t H_t)^{1-\theta} = C_t + K_{t+1} - (1-\delta)K_t$$

$$X_t = \gamma^t, \gamma \geq 1$$

Note: assume that there is a one-period nominal bond (priced in $) that pays off principal plus a nominal interest rate in the subsequent period. Include this in the budget constraint of the household.

(1-C) Write this problem as a competitive equilibrium, define the equilibrium, and solve for the first order conditions of the problem.

Suppose that $\gamma = 1$, and that the economy is in a steady state in period $t = n$. Denote steady state real variables with a superscript of "SS". Suppose that the inflation rate in this steady state is constant at the rate $\pi > 0$.

(2-C) Show that the nominal intererst rate on the bond is approxmately proportional to the inflation rate. Explain the economics behind this result.

(3-C) Show that steady state labor, consumption, and capital are all decreasing in π, and explain why.

Continue to assume the economy is in steady state. Suppose that at date $n+1$ households find out that $\gamma_{n+2} = \hat{\gamma} > 1$ permanently, but that the infinite sequence of $\{T\}_{n+1}^{\infty}$ will be unchanged.

(4-C) Show that $P_{n+1} < (1+\pi)P_n$, and that consumption, hours worked, and investment in period $n+1$ are all greater than c^{SS}, h^{ss},and i^{ss}, respectively.

Suppose that this economy had no CIA constraint, and instead was an otherwise identical optimal growth model. Suppose also that this economy was in steady state.

(5-C) Evaluate the following claim as true, false, or uncertain, and explain why. "The steady state capital stock in the monetary economy will most likely be larger than in the real economy, because money is a medium of exchange that makes purchasing consumption goods easier. "

(6-C) Suppose at date $n+1$, households observe that $\gamma_{n+2} = \hat{\gamma} > 1$, permanently in period $n+2$. What do you expect to happen at date $n+1$ to the variables c_{n+1}, h_{n+1},and i_{n+1}, compared to their previous steady state values. Explain why.

(7-C) Which economy has the larger percentage change in labor at date $n+1$, the monetary economy or the real economy? Explain why.

(8-C) In one graph, draw what you think the steady state transitions will look like in the two economies, beginning at the old steady state levels, and explain why you drew the graphs in the way that you did. (Note that your graph will include two lines, one for the transition of the capital stock in the monetary economy, and one for the purely real economy).