5. Signaling

A worker of type θ has a marginal product $m(\theta, z) = \theta$. This type's cost of education level z is $C(\theta, z) = z^2 / \theta$. A type θ worker has an outside opportunity yielding a payoff of $\underline{U}(\theta) = \theta^{1/2}$. Types are uniformly distributed on $[0, 4]$.

(a) Is there a PBE in which every type chooses the same $\hat{z}$ and the equilibrium response is the expected marginal product $\mathsf{E}[\theta] = 2$?

(b) If so, does it satisfy the Cho-Kreps Intuitive Criterion?

(c) Suppose no types signal. Is this a PBE?

(d) Is there a separating PBE in which all types signal?

(e) Is there a continuum of separating PBE in which a subset of types signal? Explain.

6. Public goods

There are two consumers. If q units of the public good are produced then consumer i's gain is the strictly increasing function $B_i(\theta_i, q)$. The cost of production is $C(q)$. The parameter θ_i, $i = 1,2$ is private information. It is continuously distributed on $\Theta = [\alpha, \beta]$

(a) For this model describe in detail the VCG net contribution mechanism in which consumers are asked to reveal their types.

(b) Show that truth-telling is a BNE strategy.

(c) Explain why truth-telling is a dominant strategy equilibrium

(d) Solve for the equilibrium payoff of the designer as a function of $\theta = (\theta_1, \theta_2)$.

(e) Suppose that $B_i(\theta_i, q) = \theta_i q$ and $C(q) = \frac{1}{2}q^2$. Is the designer profit positive for all realizations if (i) $[\alpha, \beta] = [4,6]$ (ii) $[\alpha, \beta] = [2,6]$?