1. **A Non-Transitive Consumer:** Define a relation $\succeq$ on $\mathbb{R}^2_{++}$ by

$$(x, y) \succeq (x', y') \text{ IF } x \geq y' \text{ OR } y \geq y' (\text{or BOTH})$$

   (a) Show by example that this relation $\succeq$ is NOT TRANSITIVE.

   [This implies that $\succeq$ is NOT a preference relation of the sort emphasized in class, so be careful below.]

   (b) For $(x, y) \in \mathbb{R}^2_{++}$, sketch the upper, lower and indifference sets

$$\begin{aligned} U(x,y) &= \{(x',y') : (x',y') \succeq (x,y)\} \\ L(x,y) &= \{(x',y') : (x,y) \succeq (x',y')\} \\ I(x,y) &= \{(x',y') : (x,y) \succeq (x',y') \text{ AND } (x',y') \succeq (x,y)\} \end{aligned}$$

   It will probably be best if you make three separate sketches to the same scale.

   (c) For strictly positive prices $p_x > 0, p_y > 0$ and strictly positive income $m > 0$, the budget set is

$$B(p_x, p_y; m) = \{(x, y) : p_x x + p_y y \leq m\}$$

   By definition, the bundle $(a, b) \in B(p_x, p_y; m)$ is *optimal* if there does not exist another bundle $(c, d) \in B(p_x, p_y; m)$ such that $(c, d) \succ (a, b)$ (i.e. there is no bundle $(c, d) \in B(p_x, p_y; m)$ such that $(c, d) \succeq (a, b)$ and $(a, b) \not\succeq (c, d)$. The "demand set" $D(p_x, p_y, ; m)$ is the set of optimal bundles in the budget set $B(p_x, p_y; m)$.

   Find the demand set $D(p_x, p_y, ; m)$.

   [Again: be careful because the relation $\succeq$ is not transitive.]

2. **Specialization and Trade:** Canada and Mexico can each produce electronics E and food F using their own labor L_C, L_M according to the production functions

   - Canada: $E = AL_C; F = 4L_C$
   - Mexico: $E = L_M; F = 2L_M$

   where $A > 0$ is a parameter. Electronics and Food can be traded freely between the two countries but labor is immobile (i.e. production in each country can use only labor from that country). Canada and Mexico have the same number of consumer-workers; each has 3 units of labor and has preferences $u(E, F, L) = EFL$. (E = electronics, F = food, L = labor consumed as leisure.)

   (a) For what value(s) of the parameter A is there an equilibrium of this two-country economy in which both Canada and Mexico produce both Electronics and Food?

   (b) For what value(s) of the parameter A is there an equilibrium of this two-country economy in which Canada produces only Food and Mexico produces only Electronics?

   (c) For what value(s) of the parameter A is there an equilibrium of this two-country economy in which Canada produces only Electronics and Mexico produces only Food?

3. **A War of Attrition:** Two animals can "Fight" over some prey or "Quit". Payoffs are given by the following matrix:

	F	Q
F	-1, -1	10, 0
Q	0, 10	0, 0

The interpretation is that the prey is worth 10, but that an animal gets it only if it fights and the other quits, whereas both animals loose 1 when they both fight.

(a) What are the Nash equilibria (in mixed and pure strategies) of this static game?

(b) Show that there exists a subgame-perfect equilibrium, in which animal 1 "Fights" in round 1, and animal 2 "Quits"!

(c) Find the value of $p \in [0; 1]$, such that the symmetric strategy profile in which each animal fights with probability p (and quits with probability $1 - p$) after any history (randomizing independently across histories) is a subgame perfect Nash equilibrium! (Hint: The probability p is determined by the indifference condition $u(Q, p) = 0 = u(F, p) = (1 - p)\,10 + p(-1 + \delta v)$ where v is the continuation value of playing the game again when both players Fight this round. To find this value, you can assume that in the next round you are indifferent between playing Fight and Quit (this is the one-stage-deviation-principle)).

(d) The realized payoff to each animal in this strategy profile depends on the realization of their randomizing between F and Q. What is the highest possible realized payoff to animal 1? What is the lowest?

4. **Trials and Settlements:** Two risk-neutral parties, the defendant (D) and the plaintiff (P), are involved in a dispute where the plaintiff is seeking compensation from the defendant. If the parties do not settle, there will be a trial, which will cost each party $c \in (0, 1)$, and which will result in damages of v (which the defendant will have to pay to the plaintiff). The parties may, however, settle before going to trial. Before settling, they may (at the defendant's discretion) also hold a mock trial to resolve uncertainty concerning v.

   The order of decisions is as follows.

   - First, nature selects v uniformly from $[0, 1]$; v is not revealed to either party.
   - Second, the defendant decides whether to hold a mock trial. If the defendant chooses to hold a mock trial, both the defendant and the plaintiff incur a cost $d > 0$, and both immediately learn the true value of v.
   - Third, regardless of whether a mock trial is held, the game proceeds as follows:
     - The defendant makes a settlement offer $q \geq 0$ to the plaintiff, and the plaintiff either accepts or rejects this offer.
     - If the offer is accepted, the defendant pays q to plaintiff and the game ends.
     - If the offer is rejected, the dispute is resolved at trial.

   Throughout, assume that this process consumes very little time, so that there is no discounting of future payoffs.

   (a) Draw the extensive form of this game.

   (b) Suppose the defendant chooses the mock trial. What is the subgame perfect equilibrium outcome of the bargaining process after the mock trial? (Recall that D's offer must be positive, $q \geq 0$.)

   (c) Suppose the defendant chooses to forego the mock trial. What is the outcome of the bargaining process after foregoing the mock trial, assuming that we are studying sequential equilibria? (Remember, you will need to think about the plaintiff's beliefs about nature's choice for offers not chosen in equilibrium.)

   (d) Given your answers to parts b and c, does the defendant choose the mock trial? Interpret your answer. (For the interpretation you may assume that $c > E[v] = 0.5$)