UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2015)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Economy with Quasi-Linear Preferences

Consider n consumers with the following quasi-linear preferences: consumer i's utility from consuming $(x_i, m_i) \in \mathbb{R}_+^{L-1} \times \mathbb{R}$ is given by $v_i(x_i) + m_i$ (note that m_i can be any real number). Assume that v_i is continuously differentiable, concave, strictly increasing in $\mathbb{R}_+^{L-1}$ and $\lim_{x_{i,\ell} \to 0} \frac{\partial v_i(x_i)}{\partial x_{i,\ell}} = \infty$ given any $x_{i,-\ell} \in \mathbb{R}_+^{L-2}$. An allocation $(\mathbf{x}, \mathbf{m}) \in \mathbb{R}_+^{(L-1)n} \times \mathbb{R}^n$ in this economy is feasible if $\sum_{i=1}^{n} x_i \leq r$ and $\sum_{i=1}^{n} m_i = M$, where $r \in \mathbb{R}_{++}^{L-1}$ and $M > 0$ are the total resources that are available in this economy. Answer the following questions.

(a) A feasible allocation $(\mathbf{x}, \mathbf{m})$ is Pareto efficient if and only if $\mathbf{x}$ solves the following problem:

$$(P) \quad \max_{\mathbf{x} \in \mathbb{R}_+^{(L-1)n}} \sum_{i=1}^{n} v_i(x_i) \text{ s.t. } \sum_{i=1}^{n} x_i \leq r.$$

Prove this statement in two steps.

(i) If $\mathbf{x}$ solves (P), then any feasible allocation $(\mathbf{x}, \mathbf{m})$ is Pareto efficient. Explain why this is the case briefly.

(ii) Prove the other direction. Show that, if a feasible allocation $(\mathbf{x}, \mathbf{m})$ is Pareto efficient, then $\mathbf{x}$ must solve (P).

(b) Write down the necessary and sufficient condition (Kuhn-Tucker condition) for the optimal solution for the problem (P). Explain why it is necessary and sufficient briefly.

(c) Let $\mathbf{x}^*$ be a solution for (P). Show that there exists $p^* \in \mathbb{R}_{++}^{L}$ such that $(\mathbf{x}^*, \mathbf{m}^*, p^*)$ is a Walrasian equilibrium with transfer for any $\mathbf{m}^*$ such that $\sum_{i=1}^{n} m_i^* = M$ (Do not just appeal to the second welfare theorem).

2. Insurance

Kenny is considering to purchase a car insurance. There are three possibilities: he may have no car accident, or a minor accident, or a major car accident with equal probability (Kenny is not a good driver). If he is involved with a minor accident, he would lose \$1000. If he is involved with a major accident, he would lose \$5000. An insurance is given by $(\delta, B_{\min}, B_{maj})$ where δ is the premium to pay in advance, $B_{\min}$ and B_{maj} are the benefits Kenny would receive in the case of a minor accident and a major accident respectively. He is an expected utility maximizer with a strictly increasing vNM utility function $u(\cdot)$. So his expected utility from an insurance $(\delta, B_{\min}, B_{maj})$ is $\frac{1}{3}u(-\delta)+\frac{1}{3}u(B_{\min}-\delta-1000)+\frac{1}{3}u(B_{maj}-\delta-5000)$. Answer the following questions.

(a) Consider an insurance $(\delta, B_{\min}, B_{maj}) = (200, 1000, 3000)$. Does Kenny prefer this insurance to no insurance (for any strictly increasing u)? If so, prove it. If you think that it depends on the shape of u, then find u such that Kenny prefers not to buy this insurance.

(b) Consider an insurance $(\delta, B_{\min}, B_{maj}) = (200, 1200, 1200)$. Does Kenny prefer this insurance to no insurance? If so, prove it. If you think that it depends on the shape of u, then find u such that Kenny prefers not to buy this insurance.

(c) Consider another insurance $(\delta, B_{\min}, B_{maj}) = (200, 400, 800)$. Suppose that Kenny is risk averse: u is strictly concave. Does Kenny prefer this insurance to no insurance? If so, prove it. If you think that it depends on the shape of u, then find u such that Kenny prefers not to buy this insurance.

(d) Consider the following two insurances that are fair (i.e. $-\delta+\frac{1}{3}B_{\min}+\frac{1}{3}B_{maj}=0$): $(\delta', B'_{\min}, B'_{maj}) = (100, 100, 200)$ and $(\delta'', B''_{\min}, B''_{maj}) = (100, 0, 300)$. Suppose that these insurances are divisible. If Kenny purchases $x \geqq 0$ units of the first insurance and $y \geqq 0$ units of the second, then his total insurance can be represented as $(x\delta'+y\delta'', xB'_{\min}+yB''_{\min}, xB'_{maj}+yB''_{maj})$. Suppose that u is strictly concave. Discuss how Kenny would combine these two insurances optimally.

3. Subgame Perfect Bargaining

Two players must divide \$1 according to the following procedure: Player 1 proposes a division $(x, 1-x)$ (with $0 \leq x \leq 1$); Player 2 can Accept or Reject. If Player 2 Accepts, the proposed division is implemented, otherwise both agents get 0.

Players 1 and 2 care about their own consumption and also about fairness; if the outcome is (x_1, x_2), their utilities are

$$\begin{aligned} u_1(x_1, x_2) &= x_1 - \theta_1 |x_1 - x_2| \\ u_2(x_1, x_2) &= x_2 - \theta_2 |x_1 - x_2| \end{aligned}$$

where $\theta_1, \theta_2 \geq 0$ are parameters that measure how much players care about fairness.

Find all the pure strategy subgame perfect equilibria of this game. Of course your answer will depend on the parameters θ_1, θ_2.

You may find it helpful to first graph players' utility for divisions $(x, 1-x)$ as a function of x and think about how the parameters θ_1, θ_2 affect the graph.

4 Repeated Games

For the stage game G below, consider the infinitely repeated game in which players use the discount factor $\delta \in (0, 1)$.

G

	L	R
U	3.1,1	1,3.1
D	0,2	2,0

(a) Which long term average payoffs can be supported as subgame perfect equilibria (in pure strategies) for δ very close to 1? (Provide an explicit description of these payoffs.)

(b) Which long term average payoffs can be supported as subgame perfect equilibria (in pure strategies) for $\delta = .9$? Give a complete argument.

5. Efficient Mechanism Design

Agent i's benefit if q units of a public good is produced is the strictly concave function $B_i(\theta_i, q)$. Agent i's type is continuously distributed on $\Theta = [\alpha, \beta]$. The cost of the public good is k per unit.

(a) For the two agent case, prove that if the designer uses the Net Contribution to Social Surplus mechanism (NC mechanism), then truth telling is a dominant strategy for each agent.

(b) Suppose that $B_i(\theta_i, q) = \theta_i q - \frac{1}{4}q^2$ and $2\alpha > k$. Confirm that it is always efficient to produce some of the public good.

(c) Show that as long as β is not too large relative to α, then the NC mechanism results in a profit to the designer for all possible types.

(d) Suppose instead that $2\alpha < k$ so that it is not efficient to produce any of the public good if both types are sufficiently small. Is the result above still true? Explain.

(e) Suppose that there is a single buyer with demand price function $p_1(q) = \theta_1 - \frac{1}{2}q$ and a single seller with a marginal cost function $MC_2 = \phi_2 + \frac{1}{2}q$. Show that total surplus is

$$S(\theta_1, \phi_2, q) = (\theta_1 - \phi_2) q - \frac{1}{2}q^2.$$

Use the above results to try and draw conclusions about the feasibility of an efficient mechanism that always yields the designer a profit in this example.

6. Choice of Signals

The value of a type θ worker is $m(\theta, q) = \theta$. A player's type is continuously distributed on $[0, 4]$. This worker has an outside opportunity with payoff $u_o(\theta) = 0$. The cost of signaling using educational technology t is $C_t(\theta, q) = \frac{B(q)}{A_t(\theta)}$, where $A_t(\theta)$ and $B(q)$ are both increasing and continuously differentiable. Also $B(0) = 0$.

Suppose first that there is just one technology.

(a) What is the critical factor that determines the rate at which higher types gain from signaling?

(b) Consider separating PBE. Show that for those types that signal,

$$A_t(\theta) U(\theta) = \int_0^\theta x A_t'(x)\, dx + k.$$

Henceforth suppose that $A_1(\theta) = \theta^{\frac{1}{2}}$ and $A_2(\theta) = 1 + \theta$.

(c) For each technology, characterize the best separating PBE. Which technology is better for a low type? Which for a high type?

(d) Now suppose that both educational technologies can be utilized. Show that it is best for both technologies to be used and solve for the set of types which utilize each technology.

(e) Does PBE in which all types choose technology 1 satisfy the Intuitive Criterion? How about a PBE in which all types are pooled?