UCLA Department of Economics **Spring 2015**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and each part is worth 10 points. Parts 1 and 2 have one question each, and Part 3 has two questions worth 5 points each. All questions are required. **Answer each question in a separate bluebook.**

You should turn in (at least) FOUR bluebooks, one (or more, if needed) bluebook(s) for each question.

Part 1

Consider an economy populated by a continuum of identical households with the following preferences:

$$\sum_{t=0}^{\infty} \beta^t \left(\ln c_t + A \ln l_t \right), \quad 0 < \beta < 1,$$

where c_t is consumption and l_t is leisure at date t. Households are endowed with one unit of time each period that can be used for labor or leisure. In addition, each household is endowed with k_0 units of capital in period 0 and can accumulate capital according to the law of motion

$$k_{t+1} = (1-\delta) k_t + i_t \ , \quad 0 < \delta < 1,$$

where i_t is investment at date t.

The households sells labor to a competitive firm and can work either a straight time shift of length h_1, a straight time plus overtime shift of length $h_1 + h_2$, or not at all (thus, labor is an indivisible commodity). The technology for combining capital with straight time and overtime labor to produce output (y_t) is given by

$$y_t = e^{z_t} \left(h_1 k_t^{\theta} (n_{1t} + n_{2t})^{1-\theta} + h_2 k_t^{\theta} n_{2t}^{1-\theta} \right), \quad 0 < \theta < 1,$$

where n_{1t} is the number of households working <u>only</u> straight time and n_{2t} is the number of households working straight time plus overtime. Output can be used for current consumption or investment. The technology shock, z_t, evolves according to

$$z_{t+1} = \rho z_t + \varepsilon_{t+1} \ , \quad \varepsilon_{t+1} \text{ is an i.i.d. random variable with mean 0.}$$

A. Carefully formulate the dynamic program that would be solved by a social planner that chooses capital, labor and consumption sequences to maximize a social welfare function that weights all agent's utilities equally.

B. Prove that in equilibrium the fraction of <u>employed</u> households that work overtime is a constant even when the economy is not in steady state. (Hint: Do this by simply deriving this constant.)

C. Suppose that there are moving costs that must be incurred when the number of straight-time workers is changed, $m_t = \frac{d}{2}(n_{1t} - n_{1t-1})^2$. The output available for consumption and investment is, in this case, $y_t - m_t$. Repeat part A for this case and show that statement in part C no longer holds.

D. Define a *recursive competitive equilibrium* for the model of part (A) where agents trade employment lotteries. Be sure to completely specify the problem solved by households and firms in your decentralized economy.

E. Derive an expression for the straight time hourly wage rate and the overtime wage rate as a function of the prices determined in part D. Next, derive an expression for the overtime wage premium, which is the ratio of the hourly overtime wage rate to the hourly straight-time wage rate, in terms of the parameters of the model. Under what conditions will the overtime premium be greater than one?

Part 2.

Preferences. Time is discrete $t \in \{0, 1, 2, \ldots\}$ and there is no uncertainty. The economy is populated by a unit measure of households who come in J different types indexed by $j \in \{1, \ldots, J\}$. The measure of type-j households is μ_j and, since there is a unit measure of households, we let $\sum_{j=1}^{J} \mu_j = 1$. The intertemporal utility of an household of type j is

$$\sum_{t=0}^{\infty} \beta^t \left[(1 - \alpha_j) \log (c_{j,t}) + \alpha_j \log \left(\frac{M_{j,t+1}}{P_t} \right) \right],$$

where $\beta \in (0, 1)$ and $\alpha_j \in (0, 1)$.

Endowments. Each household starts at time $t = 0$ with an identical endowment of $\bar{M}_0$ units of money, and $\bar{b}$ units of maturing one-period real government bonds (i.e., the household is entitled to receive $\bar{b}$ units of consumption from the government at time $t = 0$). Every subsequent period, a household of type j receives the endowment y_j. We denote the aggregate endowment by:

$$y = \sum_{j=1}^{J} \mu_j y_j.$$

Government. There is a government who needs to finance g units of consumption. Every period the government maintains a constant supply of one-period real bonds, $\bar{b}$, and levies constant lump-sum taxes $\tau \leq \min\{y_i\}$. The government creates or destroys money, $\bar{M}_{t+1} - \bar{M}_t$, in order to meet its budget constraint.

Notations. In what follows, we will denote by P_t the nominal price of consumption goods at time t, by r_t the *real* interest rate between t and $t+1$, and by i_t the *nominal* interest rate between t and $t+1$.

1. (3pt) Definitions

   (a) (0.5pt) State the government sequential budget constraint.

   (b) (0.5pt) State the intertemporal problem of a type-j household.

   (c) (1pt) Define a feasible allocation.

   (d) (1pt) Define a competitive equilibrium.

2. (4pt) Consider now the economy with one representative household, $J = 1$.

   (a) (0.5pt) Derive the first-order conditions of the household's intertemporal problem.

   (b) (0.5pt) Show that the real interest rate is constant.

   (c) (0.5pt) Guess that the growth rate of money is constant and equal to γ, that is:

$$\bar{M}_{t+1} - \bar{M}_t = \gamma \bar{M}_t,$$

   for all $g \geq 0$, and where γ is to be determined in equilibrium. Show that the inflation rate must also be constant and equal to γ.

   (d) (1pt) Derive the aggregate money demand.

   (e) (1.5pt) Derive the equation that determines the equilibrium inflation rate, γ. How does it depend on the exogenous parameters g, τ, $\bar{b}$, and α? Explain why.

3. (3pt) Consider now the economy with heterogenous households, $J > 1$. Guess that there exists an equilibrium with the following features: money growth rate and inflation are constant and equal to some γ, and the consumption, $c_{j,t}$, and real money holding, $M_{j,t+1}/P_t$, of a household of type j stay constant, equal to c_j and m_j respectively.

   (a) (1pt) By combining the first-order condition of the household's problem, and the household's *intertemporal* budget constraint, find expressions for c_j and m_j as a function of α_j, y_j, τ, $\beta/(1+\gamma)$, and $\bar{b} + \frac{M_0}{P_0}$.

   (b) (1pt) Let $\bar{\alpha} \equiv \sum_{j=1}^{J} \mu_j \alpha_j$, $\bar{m} = \sum_{j=1}^{J} \mu_j m_j$, and $\bar{c} = \sum_{j=1}^{J} \mu_j c_j$. Using the expression for c_j and m_j you found in the question above, calculate

$$(1-\bar{\alpha})\,\bar{m}\left(1 - \frac{\beta}{1+\gamma}\right) - \bar{\alpha}\bar{c}.$$

   (c) (1pt) Derive the aggregate money demand. How does it differ from the aggregate money demand when there is a representative agent? How does it depend on the correlation between the level of money demand, α_j, and the level of income, y_j? Explain why.

Part 3 Question 1

This question is worth 5 points

In this question, we examine the demand for money in a model with heterogeneous agents and exogenous incomplete markets.

Consider the following economy with heterogeneous agents. Time is discrete and denoted $t = 1, 2, 3, \ldots$. There is a continuum of measure one of agents. Each period each agent experiences an idiosyncratic endowment shock y_t drawn from the set $Y = \{y^1, y^2, \ldots, y^N\}$. Assume that for each agent, these endowments follow a Markov process with transition probabilities

$$\phi^{ij} = Prob(y_{t+1} = y^j | y_t = y^i).$$

Assume that the initial distribution of endowments across agents is given by the vector $\eta = (\eta^1, \eta^2, \ldots, \eta^N)'$ where η^i denotes the fraction of agents with initial endowment $y_0 = y^i$. Assume that η is a stationary distribution of the Markov process described by transition probabilities ϕ^{ij}.

Let $\pi_t(h^t)$ denote the probability as of date 0 that an agent experiences the history of endowment realizations

$$h^t = (y_0, y_1, \ldots, y_t)$$

and let $c_t(h^t)$ denote the consumption at time t of such agents. Assume that agents choose stochastic processes for consumption $\{c_t(h^t)\}_{t=0}^{\infty}$ to maximize their discounted expected utility with period utility $u(c)$ and time discount factor $\beta < 1$.

Part 1: One point Write an expression for agents' expected utility and write the feasibility constraints on the consumption allocation $\{c_t(h^t)\}_{t=0}^{\infty}$. When writing expected utility, note that agents know their first period endowment y_0, so be careful to write their expected utility conditioning on that initial endowment.

Now assume that the only asset available to agents is fiat money. Assume that at the start of the first period, each agent is endowed with M pieces of fiat money (nicely colored pieces of paper). Let the number of these pieces of paper be fixed over time. Let $\{P_t\}_{t=0}^{\infty}$ denote the price at which agents exchange money for goods at each date t. We look to give a recursive definition of a stationary equilibrium in which agents choose consumption c and money holdings m' each period to maximize the expected discounted present value of their consumption from the current period on given endowment y^i and money holdings m at the start of the period. In the stationary equilibrium, we assume that the price level P_t is constant at $P > 0$.

Part 2: One Point In a stationary equilibrium, agents who start the period with endowment y^i and money holdings m choose consumption $c(y^i, m)$ and money holdings to carry into next period $m'(y^i, m)$ to maximize the expected discounted present value of their consumption subject to the constraints

$$c(y^i, m) + \frac{1}{P} m'(y^i, m) = y^i + \frac{1}{P} m$$

and $m'(y^i, m) \geq 0$. Write a Bellman equation describing the problem that agents are solving to choose their optimal plans for consumption and money holdings $c(y^i, m)$ and $m'(y^i, m)$.

Part 3: One Point Sketch an argument that there exists a value of money holdings $\bar{m}$ such that the optimal choice of money holdings

$$m'(y^i, m) \leq \bar{m}$$

for all values of y^i and m with $m \leq \bar{m}$. In making that argument, you should make reference to the equilibrium real rate of return on money holdings in a stationary equilibrium. You should not try to give a detailed argument. Simply provide some intuition for why there should be an upper bound on agents' equilibrium money holdings.

Let the joint distribution of agents' holdings of money in real term and endowments at date t be given by $F_t(m/P_t|y^i)\eta^i$ where $F_t(m/P_t|y^i)$ is the cumulative distribution function of real money holdings for an agent conditional on that agent having endowment y^i at date t. Recall that η^i represents a stationary distribution of endowments so that this does not vary over time. We define a stationary equilibrium as a value function and decision rules for agents

$$V(y^i, m), c(y^i, m), m'(y^i, m)$$

together with a stationary distribution of real money holdings and endowments across agents implied by those decision rules $F(m/P|y^i)\eta^i$ (independent of time) and a constant price level P such that the goods market and the money market clear.

Part 4: One Point Write the market clearing conditions for the goods and money markets in a stationary equilibrium. Also provide an argument that if you double the stock on money in the economy from M to $2M$, then the price level in the stationary equilibrium also doubles from P to $2P$.

Part 5: One Point Now imagine that we are comparing stationary equilibria across two economies. In one of these economies agents have no uncertainty over their income. That is $y_t = y_0$ for all t for all agents. In the other, income is uncertain as described above. Present an argument that, in the first economy with no uncertainty over endowments, that there is no stationary equilibrium with finite, non-zero, and *constant* P (i.e. in which money is valued).

Part 3 Question 2

Neoclassical Growth and Heterogeneity

The economy is populated by a large number of households, indexed by i. Preferences are given by:

$$U_i = \sum_{t=0}^{\infty} \beta^t \frac{c_{it}^{1-\sigma_i} - 1}{1 - \sigma_i} \tag{1}$$

Each household is endowed at date 0 with k_{i0} units of capital, and the aggregate capital stock at date t is denoted as $k_t, t = 0, 1, 2, \ldots$.. Each household also is endowed with one unit of labor. Let the aggregate supply of labor be normalized to 1.

There are a large number of identical and competitive firms that operate a technology, denoted as f, which is homogeneous of degree 1 in capital and labor, and that produces output. The technology is increasing in capital and labor, and is twice continuously differentiable. Households receive income from supplying labor services and from renting capital. Assume that capital markets and labor markets are perfect.

1. Define a competitive equilibrium for this economy

2. Discuss the following statement, and be as precise as you can: "Economist A says that if the aggregate capital stock, k_0, is the steady state capital stock of this economy, then the initial distribution of wealth across households will be preserved, and the pattern of consumption inequality will mirror the pattern of capital inequality, even though preferences are different. Economist B says that for every $k_0 > 0$, each household's capital holdings will converge to the per-capita steady state capital stock because of diminishing marginal productivity of capital and because each household has the same level of human capital and the same ownership in the firms."