UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2016)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Stochastic Dominance

There are three lotteries p_1, p_2, p_3, each of which generates $-\$100$, $\$50$, or $\$200$ with the following probability respectively.

$$\begin{aligned} p_1(-100) &= 0.2,\ p_1(50) = 0.4,\ p_1(200) = 0.4 \\ p_2(-100) &= 0.1,\ p_2(50) = 0.5,\ p_2(200) = 0.4 \\ p_3(-100) &= 0,\ p_3(50) = 0.65,\ p_3(200) = 0.35 \end{aligned}$$

Answer the following questions.

(a) Find all pairs of lotteries for which one lottery is first order stochastically dominated by the other. Explain why there is no other such pair.

(b) Find all pairs of lotteries for which one lottery is second order stochastically dominated by the other. Explain why there is no other such pair.

(c) Consider another lottery $\widetilde{p}$ with the probability distribution: $\widetilde{p}(-100) = x$, $\widetilde{p}(50) = y$, $\widetilde{p}(200) = 1 - x - y$. Characterize the range of x and y in which p_1 is first order stochastically dominated by $\widetilde{p}$.

(d) Characterize the range of x and y in which p_1 is second order stochastically dominated by $\widetilde{p}$.

2. Pareto Efficiency with Quasi-Linear Preference

Consider a pure exchange economy $\mathcal{E}^{pure} = \left(\{X_i,\ \succeq_i,\ e_i\}_{i=1,2}\right)$ with two goods and two consumers, where $X_i = \mathbb{R}^2_+$ and $e_1 + e_2 \gg 0$. Consumer i's preference is represented by a quasi-linear utility function $v_i\left(x_{\,i,1}\right) + x_{i,2}$, where v_i is a differentiable, increasing and strictly concave function.

(a) Define Pareto efficiency and show that an allocation $x = (x_1, x_2) \in \mathbb{R}^4_+$ is Pareto-efficient if and only if there exists $\overline{u}_2$ such that x solves the following problem:

$$\begin{aligned} &\max_{x_i \geq 0} v_1\left(x_{\,1,1}\right) + x_{1,2} \\ \text{s.t. }\ & v_2\left(x_{\,2,1}\right) + x_{2,2} \geq \overline{u}_2 \\ & \sum_{i=1}^{2} x_{\,i} \leq \sum_{i=1}^{2} e_{\,i}. \end{aligned}$$

(b) Write down the Kuhn-Tucker conditions for the problem in (a) and discuss briefly why they are necessary and sufficient for the optimal solutions (assume that both consumers consume a positive amount of good 1).

Note: For the next two questions (c) and (d), you can provide a graphical answer using the Edgeworth box . But it needs to be accompanied with a clear enough explanation.

(c) Show that, if x' and x'' are interior Pareto efficient allocations in $\mathbb{R}^4_{++}$, then $x'_{i,1} = x''_{i,1}$ for $i = 1, 2$. That is, each consumer consumes exactly the same amount of good 1 across all interior Pareto-efficient allocations.

(d) Suppose that $v_1\left(x_{\,1,1}\right) = \log x_{1,1}$, $v_2\left(x_{\,2,1}\right) = 2\log x_{2,1}$, and $e_1 = e_2 = (1,1)$. Find all Pareto efficient allocations and depict them in the Edgeworth box.

3. Cournot Competition with Capacity

Two firms produce an identical good for sale in a single market. Both firms have 0 fixed cost and 0 marginal cost. The market inverse demand function is

$$P = 1 - Q$$

Firms choose quantities and the market determines the price.

(a) Suppose first that capacity is *exogenous*: firm i has capacity $k_i \in [0, .5]$. If firms simultaneously choose actual quantities (subject to their capacity constraint), find the pure strategy NE of the one-shot game. [Suggestion: It may be easier to analyze case-by-case.]

(b) Now suppose capacity is *endogenous*. Firms play a two-stage game. In the first stage, firms simultaneously choose capacities $k_i \in [0, .5]$; between stages, capacity choices are revealed; in the second stage firms simultaneously choose quantities (subject to their capacity constraint). Find all the SGPE of the two-stage game.

4 Repeated Differentiated Commodities

Two firms produce different goods for sale in a single market. Each firm's production imposes a negative externality on the other: if the firms produce quantities q_1, q_2 then their profits will be

$$\begin{aligned}\Pi_1(q_1, q_2) &= (120 - q_2)q_1 - q_1^2 \\ \Pi_2(q_1, q_2) &= (120 - q_1)q_2 - q_2^2\end{aligned}$$

(a) Suppose first that the firms interact only once. Find the (pure strategy) Nash equilibrium of the game and the (symmetric) Pareto optimum.

(b) Now suppose the firms interact infinitely often and discount future profits with the discount factor $\delta < 1$.

(i) For what values of δ (if any) is there a SGPE in which firms play the (symmetric) Pareto optimum in each period in which there has been no deviation and firms punish any deviation by permanent reversion to the one shot Nash equilibrium?

(ii) For what values of δ (if any) is there a SGPE in which firms play the (symmetric) Pareto optimum in each period in which there has been no deviation and punish *any* deviation by playing *one round* of min-max against the deviator? (After punishment is complete and there has been no deviation from punishment play returns to the (symmetric) Pareto optimum).

5. Auctioning Two Identical Items

There are two identical items for sale and three bidders. Bidder i has a value θ_i that is an independent draw from a distribution with support $[0,1]$, p.d.f. $f(\theta)$ and c.d.f. $F(\theta)$. Each bidder wishes to purchase only one item. The items are awarded to the bidders submitting the two highest bids.

(a) Suppose first that the winning bidders must pay their own bids. Explain why the equilibrium win probability is $W(\theta) = 1 - (1 - F(\theta))^2$. Solve for the equilibrium bid function if values are uniformly distributed.

Suppose, henceforth, that each of the winning bidders must pay the second highest bid. Let $B(\theta)$ be the equilibrium bid function. Suppose buyer 1 deviates and bids $B(x)$ rather than $B(\theta)$. Define $y(\theta) \equiv F(\theta) B(\theta)$.

(b) Explain carefully why buyer 1's expected payoff, if his value is θ, can be written as follows:

$$\begin{aligned} u(\theta, x) &= \theta\left(1 - (1 - F(x))^2\right) - 2(1 - F(x)) F(x) B(x) - 2\int_0^x f(z) F(z) B(z)\, dz \\ &= \theta\left(1 - (1 - F(x))^2\right) - 2(1 - F(x)) y(x) - 2\int_0^x f(z) y(z)\, dz. \end{aligned}$$

(c) Characterize the FOC for incentive compatibility as a differential equation for $y(\theta)$.

HINT: Two of the terms should cancel each other out.

(d) Solve for the equilibrium bid function.

(e) Which of the two auctions would be better for the buyers? Explain briefly (no math).

6. Signaling

Consider the basic Spencian signaling model. A type θ worker with education level z has a marginal product $m(\theta, z) = \theta$. The cost of this education is $C(\theta, z) = \frac{z}{\theta^2}$. Firms observe the education of each potential employee but not the worker's type or marginal product. Types are continuously distributed on $\Theta = [0, 2]$ with c.d.f. $F(\theta) = \frac{\theta}{2}$. The outside opportunity wage of a type θ worker is $u_o(\theta)$.

(a) Suppose that there exists a PBE which is separating on an interval $S \subset \Theta$. Show that the equilibrium utility of the types in S can be written as follows:

$$U(\theta, K) = a\theta + \frac{K}{\theta^b}.$$

For (b) and (c) assume that $u_o(\theta) = \frac{3}{4}\theta$.

(b) Fully characterize the set of separating PBE (if any).

(c) Is there a pooling PBE in which all types above $\widehat{\theta} = 1$ choose the same signal? Explain.

(d) Suppose instead that $u_o(\theta) = \frac{1}{4}\theta$. Fully characterize the set of separating PBE (if any).

(e) Do any of the PBE in (b) and (d) satisfy the Cho-Kreps Intuitive Criterion? Explain carefully.