UCLA Department of Economics **Fall 2016**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each part in a separate bluebook.** All three parts will receive equal weight in your grade.

Part I

Consider a real business cycles model with a representative household that lives forever and maximizes the following utility function:

$$E\sum_{t=0}^{\infty}\beta^t\left\{\log c_t + A\log L_t\right\},\ 0<\beta<1 \text{ and } A>0.$$

Here, c_t is consumption and L_t is a convex combination of leisure in periods t and $t-1$. Each period, households are assumed to have one unit of time that can be allocated between market work, h_t, and leisure. In particular, let $L_t = a(1-h_t)+(1-a)(1-h_{t-1})$ where $0\le a\le 1$.

Output, which can be used for consumption, investment (i_t) or government purchases, is produced according to a constant returns to scale technology, $y_t = e^{z_t}k_t^{\theta}h_t^{1-\theta}$, where y_t is output and k_t is the stock of capital. The variable z_t is a technology shock observed at the beginning of period t that evolves through time according to a first order autoregressive process with mean zero innovations. The stock of capital is assumed to depreciate at the rate δ each period.

Investment in period t becomes productive capital one period later, $k_{t+1}=(1-\delta)k_t+i_t$. Government spending is an exogenous random variable that, like the technology shock, follows a first order autoregressive process, in this case with an unconditional mean of $\bar{g}$ and unconditional variance of σ_g^2. Innovations to this process are assumed to be independent of innovations to the technology shock process and the value of g_t is observed at the beginning of period t. In addition, government purchases are ***financed with lump sum taxes***. Note that government purchases do not directly affect preferences or the technology; they are simply thrown into the sea.

(a) Are the equilibrium allocations for this economy the solution to a social planner's problem? Explain. If so, write the social planners problem for this economy as a dynamic programming problem. Be specific about the stochastic process (law of motion) for z_t and g_t.

(b) Derive as set of equations that characterize a sequence $\left\{c_t, h_t, L_t, k_{t+1}, y_t\right\}_{t=0}^{\infty}$ that solves this problem. Be sure that you have the same number of equations as unknowns. Explain the role of the *transversality condition* in determining this optimal sequence.

(c) Define a *recursive competitive equilibrium* for this economy.

(d) Assuming that for a given variable x, $\tilde{x}_t \equiv \log x_t - \log\bar{x}$, where $\bar{x}$ is the non-stochastic steady state value of x_t. Derive a linear expression for $\tilde{h}_t$ as a function of $\tilde{k}_t, \tilde{c}_t, z_t,$ and $\tilde{h}_{t-1}$.

(e) As in part (d), derive a linear equation expressing $\tilde{c}_t$ as a function of $\tilde{k}_t, z_t, \tilde{g}_t, \tilde{h}_t$, and $\tilde{k}_{t+1}$.

(f) In a standard real business cycle model, $a = 1$. Using the equation derived in part (d) and/or (e), explain how setting $a < 1$ might change the cyclical properties of the model economy. In particular, focus on the size of fluctuations in hours worked relative to z_t. Provide intuition in your explanation.

(g) In a standard real business cycle model, g_t is not included as a stochastic shock. Discuss how adding this feature might change the cyclical properties of the model economy. In particular, focus on the correlation between hours worked and z_t. Again, provide intuition.

Part 2.

1. Consider first the McCall Model we studied in class. There is an infinitely-lived risk neutral worker with discount factor $\beta \in (0,1)$. The worker can be in either one of two states, unemployed ("U") or employed ("E"). Every period when she is unemployed, she receives the unemployment benefit $b > 0$ and draws exactly one wage offer from the CDF $F(w)$. If she accepts the offer, she starts working next period at that wage. At the end of every period of employment, the worker looses her job with probability $\delta \in (0,1)$ and becomes unemployed. Let $V_E(w)$ denote the maximum attainable utility of an employed worker, and V_U the maximum attainable utility of an unemployed worker.

   (a) Write the Bellman equations for an unemployed worker, V_U, and for an employed worker at wage w, $V_E(w)$ (1pt).

   (b) Show that the optimal policy of the unemployed worker is to accept any offer above some reservation wage (1pt).

   (c) Find an expression for the reservation wage in terms of V_U (1pt).

   (d) Consider a worker who has received an offer just equal to the reservation wage. Show that, for this worker, it would be weakly optimal to accept the offer and quit after just one period of employment. (1pt)

2. Now assume that job offers are heterogenous not only in terms of wage, but also in terms of stability. That is, when receiving an offer, a worker draws both a wage w and a job destruction rate δ. A higher δ thus corresponds to a less stable job. Assume for simplicity that w and δ are independently distributed with respective CDF $F(w)$ and $G(\delta)$.

   (a) Write the Bellman equations for an unemployed worker, V_U, and for an employed worker at wage w facing job destruction rate δ, $V_E(w, \delta)$. (1pt)

   (b) Show that the reservation wage is the same regardless of the job destruction rate δ. Explain why. *Hint:* use the insight of question 1.d. (1pt)

   (c) What is the impact of an increase in δ on $V_E(w, \delta)$? Show that there are two effects going in opposite directions. Explain these two effects. Explain when each of the effect dominate and why. (1pt)

3. Now assume that workers accumulate human capital on the job. Precisely, consider an employed worker with human capital level h. If she keeps her job next period, then her level of human capital is $h'(1+\gamma)$, for some small positive γ. Assume as well that wage offers, w, are "per unit of human capital": that is a worker with human capital h and wage w receives the pay $w \times h$. Likewise, the unemployment benefit b is per-unit of human capital: a worker with benefit b and human capital h receives $b \times h$.

   (a) Write the Bellman equation for an unemployed worker with human capital h, $V_U(h)$ and for an employed worker with human capital h, wage w, and facing job-destruction rate δ, $V_E(h, w, \delta)$. (1pt)

   (b) Argue that $V_U(h) = h \times v_U$ and $V_E(h, w, \delta) = h \times v_E(w, \delta)$. Write the Bellman equation for v_U and $v_E(w, \delta)$.

   (c) Does the insight of question 1.d continue to hold? Why? (1pt)

   (d) How does the reservation wage depend on the job-destruction rate, δ? Why? (1pt)

   (e) What is the impact of an increase in γ on the value of an unemployed worker, v_U? (1pt)

   (f) Shows that an increase in γ has two impacts on the reservation wage going in opposite directions. Explain why. (1pt)

   (g) Argue that one effect always dominate for $\delta \neq 1$. Argue that the other effect can dominate for $\delta \neq 0$. (1pt)

Part 3 - Taxation and Economic Activity in an Optimal Growth Model

Preferences for the representative household, which has one unit of time available per period, are given by:

$$\max \sum \beta^t \{\ln(c_t) - \phi h_t\} \tag{1}$$

The aggregate production technology is given by:

$$AK_t^\theta (X_t H_t)^{1-\theta} \tag{2}$$

The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{3}$$

The technology process X is given by:

$$X_{t+1} = (1+\gamma)X_t \tag{4}$$

A. Explain why you can - or cannot - solve for the competitive equilibrium allocations by solving a social planning problem. **(2 points)**

B. Derive equations that can be used to solve for the planner's allocations in a stationary version of this economy. Show all of your work **(5 points)**

C. Show the equations that characterize the steady state of the planner's problem. **(3 points)**

Suppose that the economy is in steady state at date j. Suppose that from date j onwards that the government will take g units of resources every period, and transfer those resources back to the household as a lump sum transfer. Suppose that the government has access to the following taxes to obtain these resources: consumption taxes, labor income taxes, or it can tax the capital stock.

D. Show that for a sufficiently small per-period g that there exists a tax system such that this economy remains in its original steady state for all future periods, and that welfare is unaffected by the government's tax-transfer policy. Note: a tax system is defined as infinite sequences of tax rates on consumption, labor income, and the capital stock. These tax rates can be negative, zero, or positive. Show a formula that determines the maximum size of g that can be financed with this welfare-preserving tax system, and call this g^*. **(7 points)** .

E. Is your tax system equivalent to a lump sum tax system? Why or why not? **(3 points)**