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Comprehensive Examination

Quantitative Methods

Answer Questions 1 and 2 in Part I, Questions 1, 2, and 3 in Part II, and Questions 1, 2, 3, and 4 in Part III. Good luck!

Part I (based on Econ 203a)

Question 1: In a given population, individuals have to perform a given task that requires two steps. The time that it takes to an individual with characteristics Z to perform the task is $X+Y$, where X is the time for the first step and Y is the time for the second step. The joint density of (Y,X), for an individual with characteristics $Z=z$, where $z>0$, is given by

$$\begin{aligned} f_{Y,X|Z=z}(y,x) &= c\, e^{-(x+y)/z} && \textit{if} \quad y>0,\ x>0 \\ &= 0 && \textit{otherwise} \end{aligned}$$

Denote the marginal density of z by $f_Z(z)$; $f_Z(z)>0$ when $z>0$ and $f_Z(z)=0$ otherwise.

(a) What is the value of the constant c, for an individual with characteristics $Z=z$ $(z>0)$? Explain.

(b) Are X and Y independent, conditional on $Z=z$? Explain.

(c) Let $H=X+Y$. Derive the density of H, conditional on Z.

(d) Calculate the expectation and variance of H, conditional on $Z=z$, when $z>0$.

(e) Can you use your answer to (b) to determine whether X and Y are independent? Explain in detail how you would determine whether X and Y are independent.

(f) Let $W_1=\min\{X,Y\}$ and $W_2=\max\{X,Y\}$. Determine the distribution of (W_1,W_2) conditional on Z.

Question 2: The value, V, that a particular individual derives from a particular work is given by

$$V = W + m(X) + \beta E + \varepsilon$$

where W is the net income derived from the work, which may be positive or negative, X is a vector of characteristics of the work and of the individual, E is experience, and ε is an unobserved random variable. Let $Y = 1$ if the individual works and $Y = 0$ otherwise. Assume that an individual works if $V \geq 0$. Otherwise, he/she does not work. Assume that the function m and the value of the parameter β are unknown. Assume also that ε is distributed independently of (W, X, E), with a continuous, strictly increasing distribution F_ε, and that (W, X, E) has an everywhere positive density.

(a) Derive the conditional probability that an individual with $(W, X, E) = (w, x, e)$ works, as a function of (w, x, e) and the unknown functions, parameter, and distribution.

(b) Suppose that it is known that at some value x^* of X, $m(x^*) = 0$. Can you identify $(m, \beta, F_\varepsilon)$ from the distribution of Y given (W, X, E)? If your answer is yes, prove it. If your answer is no, specify additional conditions under which you can identify $(m, \beta, F_\varepsilon)$, and prove that with those additional conditions, $(m, \beta, F_\varepsilon)$ is identified.

(c) Suppose now that the value of W for any particular individual is unobserved. In particular, assume that the only observable distributions are the distribution of Y conditional on (X, E) and the distribution of W conditional on (X, E). Suppose that the distribution of W conditional on (X, E) is discrete with only two points of support, w^H and w^L. Denote the probability that $W = w^H$ conditional on $(X, E) = (x, e)$ by $p^H_{x,e}$. What can you say about the probability that an individual with $(W, X, E) = \left(w^H, x, e\right)$ works? Explain.

Part II (based on Econ 203b)

Question 1:

Suppose that you are given a linear regression model

$$y_i = x_i\beta + \varepsilon_i$$

where $E\left[z_i\varepsilon_i\right] = 0$. You do not observe the triplet (y_i, x_i, z_i) in any data set. You do have the following two data sets though:

1. (y_i, z_i) iid $i = 1, \ldots, n_1$
2. (x_j, z_j) iid $j = 1, \ldots, n_2$

Assume that $n_1, n_2 \to \infty$, and construct a consistent estimator of β. Prove why your estimator is consistent. In order to distinguish the three data sets, write $\left(y_i^{(1)}, z_i^{(1)}\right)$ for observations in the first data set, and $\left(x_j^{(2)}, z_j^{(2)}\right)$ for observations in the second data set.

Question 2:

Suppose that we are given iid (x_i, u_i) $i = 1, \ldots, n$. We have

$$(x_i, u_i)' \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}\right).$$

We observe (x_i, y_i) $i = 1, \ldots, n$ such that

$$y_i = x_i\beta + \varepsilon_i$$

where $\varepsilon_i = x_i u_i$. Let

$$\widehat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$$

What is the asymptotic distribution of $\sqrt{n}\left(\widehat{\beta} - \beta\right)$? You are expected to provide a numerical characterization of the asymptotic distribution. If you simply provide an analytic formula, you will be given no credit whatsoever.

Question 3:

Consider a simple binary response model

$$y_i = \begin{cases} 1 & \text{if } \beta - \varepsilon_i \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

where $\varepsilon_i \sim N(0,1)$. Calculate the Fisher information for β, and use it to characterize the asymptotic variance of the MLE $\widehat{\beta}$ for β.

Now, consider another estimator

$$\widetilde{\beta} = \Phi^{-1}(\overline{y})$$

for β, where $\overline{y}$ denotes the sample average of y_i and Φ^{-1} denotes the inverse of the CDF of $N(0,1)$. Prove that $\widetilde{\beta}$ is consistent and asymptotically normal. Also derive the asymptotic variance of $\widetilde{\beta}$ using the Delta method. How does the asymptotic variance of $\widetilde{\beta}$ compare with that of $\widehat{\beta}$? Hint: The derivative of Φ^{-1} is given by the expression

$$\frac{d\Phi^{-1}(x)}{dx} = \frac{1}{\phi\left(\Phi^{-1}(x)\right)}$$

where ϕ denotes the inverse of the CDF of $N(0,1)$.

Part III (based on Econ 203c)

Question 1: True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

a) In general, in HAC estimation, the bandwidth S_T has to grow to infinity as the sample size T goes to infinity to guarantee that the variance of the HAC estimator converges to zero.

b) In the standard linear regression model with errors distributed as t with v degrees of freedom, OLS is more efficient than LAD.

c) If $X_n = O_p(n^\delta)$ for some $\delta > 0$ it can not be the case that $X_n = o_p(1)$.

d) In a linear regression model with omitted regressors, OLS estimation is inconsistent.

Question 2: Take the linear model $y_i = x_i\beta + e_i$, $E(e_i|x_i) = 0$, where x_i and β are scalars.

a) Show that $E(e_i x_i) = 0$ and $E(e_i x_i^2) = 0$.

b) Is $z_i = (x_i, x_i^2)$ a valid instrumental variable for estimation of β?

c) Write down the formula for the 2SLS estimator of β using z_i as an instrument for x_i (the formula of the 2SLS estimator can always be written down even if the instruments were invalid). Do 2SLS and OLS differ here?

d) Find the efficient GMM estimator of β based on the moment condition $E(z_i(y_i - x_i\beta)) = 0$. Does it differ from 2SLS and/or OLS?

Question 3: Consider the linear IV model $y_i = x_i\beta + u_i$, $x_i = Z_i'\pi + v_i$, where $\beta \in R$, $Z_i \in R^k$, $EZ_iv_i = 0$, and $Eu_i = Ev_i = 0$.

a) The goal is to construct a confidence interval (CI) for β. Derive the asymptotic distribution of the statistic

$$AR(\beta) = n^{-1} \sum_{i=1}^{n} g_i(\beta)'[n^{-1} \sum_{i=1}^{n} g_i(\beta) g_i(\beta)']^{-1} \sum_{i=1}^{n} g_i(\beta), \tag{1}$$

where $g_i(\beta) = Z_i(y_i - x_i\beta)$ under both i) $\pi \neq 0$ fixed and ii) $\pi = \pi_n = h/n^{1/2}$ for a fixed $h \in R^k$. Describe then how you would use this statistic to construct a CI for β. Intuitively speaking, what are weak instruments? Explain why this CI has asymptotic coverage probabilities that are not affected by weak instruments.

b) Assume now conditional homoskedasticity and $k = 1$. Derive the asymptotic distribution of the 2SLS estimator when as $n \to \infty$ we have $\pi = \pi_n = h/n^{1/2}$. Explain why your result sheds light on the fact that in Monte Carlo simulations we found that a t test may overreject the true null hypothesis $H_0 : \beta = \beta_0$ when instruments are weak.

Question 4: In the context of an ARMA(p, q) model, define what we mean by a causal solution. For the linear difference equation $y_t = y_{t-1} + u_t$ with u_t iid $N(0, \sigma^2)$, is there a causal solution?