Department of Economics **Fall 2010**
UCLA

Comprehensive Examination

Quantitative Methods

Answer all questions. Question 1 if from 203a. Questions 2, 3, and 4 are from 203b. Questions 5 and 6 are from 203c. Good luck!

Question 1:

The distribution of a random variable Y conditional on a random variable X is Normal with mean $\mu(X) + \varepsilon$ and variance $\sigma(X)^2$ where μ and σ are continuous function and ε is a random variable distributed independently of X with a $N(c, \omega^2)$ distribution. The random variable X is such that

$$X = \begin{cases} -1 & \text{with probability } p_1 \\ 0 & \text{with probability } p_2 \\ 1 & \text{with probability } 1 - p_1 - p_2 \end{cases}$$

(a.1) Obtain an expression, as simple as you can, for the marginal distribution of Y. What are the expectation and variance of Y?

(a.2) Derive an expression for the Moment Generating Function of Y.

(a.3) Obtain an expression, as simple as you can, for the probability that $X = 1$ given that $Y < 0$. Is ε independent of X conditional on $Y < 0$? Explain.

(b) Suppose that you have a random sample $\{Y_i, X_i\}_{i=1}^N$ Assume that $\mu(0) = 0$ and that $\omega = 1$. Can you provide consistent, closed form, estimators for p_1 and p_2, $\mu(-1)$, $\mu(1)$, c, $\sigma(-1)$, $\sigma(0)$, and $\sigma(1)$? If your answer is yes, derive the estimators and show that they are consistent. If your answer is no, explain, determine which of the parameters can be estimated consistently and for those parameters, specify estimators and show they are consistent.

(c) Suppose now that the model is such that the distribution of Y conditional on X has mean $\mu(X) + \varepsilon$ and variance $\sigma(X)^2$, but it is otherwise unknown. Further, suppose that X is a continuous random variable, with unknown density f_X. The continuous distribution of ε is also unknown. Assume as above that the unobserved ε is distributed independently of X and that (Y, X) is observed. Determine what means, variances, and distributions can be identified in this model. Explain with as much detail as possible.

Question 2

You are given a following linear regression model

$$y_i = \alpha + \beta x_i + \varepsilon_i, \quad i = 1, \ldots, n$$

where $(x_i, \varepsilon_i)'$ is an iid random vector such that x_i and ε_i are independent of each other and $E[\varepsilon_i] = 0$. Suggest an (approximate) 95% confidence interval of β, and provide a rigorous asymptotic justification.

Question 3

You are given a following linear regression model

$$y_i = \beta x_i + \gamma z_i + \varepsilon_i, \quad i = 1, \ldots, n$$

where $(x_i, z_i, \varepsilon_i)'$ is an iid random vector such that $(x_i, z_i)'$ and ε_i are independent of each other and $E[\varepsilon_i] = 0$. What is the asymptotic variance of $\widehat{\beta}$ obtained in the OLS of y_i on (x_i, z_i). Now, assume that γ is known and consider the OLS of $y_i - \gamma z_i$ on x_i. Call the resulting OLS estimator $\widetilde{\beta}$. What is the asymptotic variance of $\widetilde{\beta}$? Is it possible that the two asymptotic variances are equal to each other? What condition needs to be satisfied if it were to be the case?

Question 4

Consider the following two-equation model:

$$y_1 = \gamma_1 y_2 + \beta_{11} x_1 + \beta_{21} x_2 + \beta_{31} x_3 + \varepsilon_1$$
$$y_2 = \gamma_2 y_1 + \beta_{12} x_1 + \beta_{22} x_2 + \beta_{32} x_3 + \varepsilon_2$$

We assume that $E[\varepsilon_1 x_1] = E[\varepsilon_1 x_2] = E[\varepsilon_1 x_3] = E[\varepsilon_2 x_1] = E[\varepsilon_2 x_2] = E[\varepsilon_2 x_3] = 0$. We know that $\beta_{21} = \beta_{32} = 0$. Establish whether the parameters are identified. For identified parameters, propose a consistent estimator, and prove why it is consistent.

Question 5

You are given a following linear regression model

$$y_i = \theta x_i + \varepsilon_i, \quad i = 1, \ldots, n$$

where $(x_i, \varepsilon_i)'$ is an iid random vector such that x_i and ε_i are independent of each other. It is known that $\varepsilon_i \sim N(0, 1)$.

1. Discuss in detail the best test for testing $H_0 : \theta = 1$ against $H_1 : \theta = 2$.
2. Derive the maximum likelihood estimator $\widehat{\theta}$ for θ.
3. Derive a consistent estimator of the asymptotic variance of $\sqrt{n}\left(\widehat{\theta} - \theta\right)$.
4. Discuss in detail the LR test for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$. Here, the LR test denotes the asymptotic version of the test. You should derive the LR test statistic in detail. Reproducing the general formula will result in zero credit.
5. Discuss in detail the Wald test for testing $H_0 : \theta = 1$ against $H_1 : \theta \neq 1$.

Question 6

6. Consider the multinomial choice model

$$y_{ij} = I(w_{ij}'\theta + \epsilon_{ij} > w_{ik}'\theta + \epsilon_{ik} \quad \forall k \neq j)$$

   Describe intuitively why the assumption that ϵ_{ij} is independent across j might be an overly restrictive assumption.

7. Consider the panel data discrete (binary) choice model

$$y_{it} = I\left(w_{it}'\theta + \sigma\alpha_i + \epsilon_{it} > 0\right)$$

   where ϵ_{it} are standard "logit errors" (i.e. with CDF $F(z) = \frac{\exp(z)}{1+\exp(z)}$) that are i.i.d. across $i = 1, ..., N$ and $t = 1, ..., T$, and where α_i has a $N(0, 1)$ distribution. Suppose ϵ_{it} and α_i are independent of each other, and independent of the w_{it}'s. Write down a simulated log-likelihood function for the dataset that is smooth in the parameter vector (θ, σ).