Department of Economics **Fall 2009**
UCLA

Comprehensive Examination

Quantitative Methods

Answer all questions. Good luck!

Part I (based on Econ 203a)

Question 1 (100 points):

Two teachers from a population of teachers are asked to evaluate students in a population of students using grades in a scale from 1 (bottom) to 3 (top). Let x^i denote the vector of observable characteristics of student i and z_j denote the vector of observable characteristics of teacher j. The valuation of teacher j for student i is given by

$$V_j^i = m\left(x^i\right) + \varepsilon^i + \eta_j \qquad j = 1, 2$$

where m is an unknown function, and ε^i and η_j are unobservable. Let t_j^i denote the grade of student i from teacher j. Assume that

$$t_j^i = \left\{ \begin{array}{lll} 1 & & if \quad V_j^i \leq \alpha_1 \\ \\ 2 & if & \alpha_1 < V_j^i \leq \alpha_2 \\ \\ 3 & & if \quad \alpha_2 < V_j^i \end{array} \right\}$$

where α_1 and α_2 are unknown parameters.

Suppose that (ε^i, x^i) is distributed independently across i, and it is independent of $\left(\eta_j, z_j\right)$ for all j. The vector $\left(\eta_j, z_j\right)$ is independent across j.. Let $F_{(\varepsilon,x)}$ and $F_{(\eta,z)}$ denote the known distributions of (ε, x) and (η, z) . The observable variables are t_1^i, t_2^i, x^i and the observable characteristics, z_1, z_2 of the two teachers.

(a) Obtain an expression, in terms of the known and unknown parameters, functions, and distributions, for the probability that $t_1^i = 1$ conditional on x^i.

(b) Obtain an expression, in terms of the known and unknown parameters, functions, and distributions, for the probability that $t_1^i = 1$ and $t_2^i = 1$ conditional on x^i.

(c) Obtain an expression, in terms of the known and unknown parameters, functions, and distributions, for the probability that $t_2^i = 1$ conditional on $t_1^i = 1$ and x^i.

(d) If you could observe only the probability that $t_1^i = 1$ given $t_2^i = 1$ and x^i. What could you say about the probability that $t_1^i = 2$ given $t_2^i = 1$ and x^i?

(e) Determine what parameters and functions, or combinations of them, are identified from the distribution of the observable variables.

(f) State a set of minimal conditions under which the function m is identified.

Suppose that conditional on x^i, ε^i is distributed $N(\beta x^i, \sigma^2)$ where β and σ are unknown parameters; conditional on z_j, η_j is distributed $N(\gamma z_j, \omega^2)$, where γ and ω are unknown parameters; z_j is $N(0, I_z)$ and x^i is $N(0, I_x)$, where I_z and I_x are unity matrices.

(g) Obtain expressions, in terms of the known and unknown parameters, functions, and distributions, for the probability that $t^i = 1$, conditional on x^i, z_j, and for the probability that $t^i = 1$ conditional on x^i.

(h) Determine what parameters and functions, or combinations of them, are identified from the distribution of the observable variables, and state a set of minimal conditions under which the function m is identified.

Part II (based on Econ 203b)

Question 1:

Consider the linear model given by

$$y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where x_i are **non-stochastic**, and the econometrician knows that

$$\epsilon_i \sim N(0,1) \quad i.i.d.$$

Let

$$\widehat{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n x_i^2}$$

Prove that

$$\Pr\left[\widehat{\beta} - \frac{1.96}{\sum_{i=1}^n x_i^2} \leq \beta \leq \widehat{\beta} + \frac{1.96}{\sum_{i=1}^n x_i^2}\right] = 95\%$$

Now consider a variant of the model where x_i are **stochastic**, and ϵ_i is independent of x_i. The econometrician knows that $E[\epsilon_i] = 0$ and $\mathrm{Var}(\epsilon_i) = 1$, but does not know the PDF of ϵ_i. Can we still say that

$$\Pr\left[\widehat{\beta} - \frac{1.96}{\sum_{i=1}^n x_i^2} \leq \beta \leq \widehat{\beta} + \frac{1.96}{\sum_{i=1}^n x_i^2}\right] = 95\%?$$

Can we say that

$$\Pr\left[\widehat{\beta} - \frac{1.96}{\sum_{i=1}^n x_i^2} \leq \beta \leq \widehat{\beta} + \frac{1.96}{\sum_{i=1}^n x_i^2}\right] \to 95\%$$

as $n \to \infty$? Under what conditions is this limiting statement true?

Question 2:

Consider the linear regression model

$$y_i = \beta x_i + \epsilon_i, \quad i = 1, \ldots, n,$$

where (x_i, ε_i) $i = 1, \ldots, n$ are iid. We assume that x_i is independent of ε_i. We further assume that $E[\epsilon_i] = 0$ and $\mathrm{Var}(\epsilon_i) = 1$, but we do not assume anything on the PDF of ϵ_i. It is known that the β is related to some economic parameter α through the relation

$$\beta = \frac{\alpha}{1-\alpha}$$

Construct a 95% confidence interval of α that is asymptotically valid.

Question 3:

Suppose that $Z_i \overset{iid}{\sim} N\left(\theta^3, 1\right)$. Compute the Fisher Information for θ when the sample size is equal to 1. Evaluate the information when θ happens to be equal to 0.

Now, prove that the maximum likelihood estimator $\widehat{\theta}$ for θ is such that

$$\widehat{\theta} = \left(\overline{Z}\right)^{\frac{1}{3}}$$

where $\overline{Z} = \frac{1}{n}\sum_{i=1}^{n} Z_i$. Prove that $\widehat{\theta}$ consistent for θ.

Assuming that the true value of θ happens to be equal to 0, calculate the asymptotic variance of $\sqrt{n}\left(\widehat{\theta} - \theta\right)$ predicted by the delta method. Do you get a reasonable number? Try to resolve the puzzle by exploiting the Fisher Information you calculated above.

Part III (based on Econ 203c)

Question 1) True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

(i) In a linear regression model with iid data, HAC estimation is consistent, only if the bandwidth S_T grows to infinity when $T \to \infty$.

(ii) With an MSE loss function, the optimal linear forecast of Y_t based on $(Y_{t-1}, ..., Y_1)$ does not depend on third and higher moments of the data only if the data are Gaussian.

(iii) In a multiple testing setup, Bonferroni's procedure does not reject the null hypothesis more often than Holm's procedure.

(iv) When instruments in the linear instrumental variables model are potentially weak, one cannot construct a confidence region for the structural parameter vector that has correct asymptotic coverage probability.

Question 2)

(i) If a causal AR(1) process is observed every other time period (i.e. instead of $y_1, y_2, y_3, ...$ you observe $y_1, y_3, y_5, ...$), is the observed process still AR(1)? If yes, express the parameters of the new process in terms of those of the original process, if no, prove your claim.

(ii) For a covariance stationary process Y_t derive a formula [in terms of $\mu \equiv E(Y_t)$, γ_0, γ_1, and γ_2, where γ_k denotes the covariance of Y_t at lag k] for the linear projection of Y_{t+1} on a constant and Y_{t-1}. Using that result, calculate this linear projection if Y_t is an AR(1) process given by $Y_t = c + \phi Y_{t-1} + \varepsilon_t$.

Question 3) **(i)** You want to test $H_0 : \theta = 0$ against $H_1 : \theta > 0$. Reject H_0 if $T_n \equiv n^{1/2}\widehat{\theta}/s(\widehat{\theta}) > q_n^*(1-\alpha)$, where the critical value is defined as follows. Using the non-parametric bootstrap, you generate B bootstrap samples, calculate the B estimates $\widehat{\theta}^*$ on these samples and then calculate $T_n^* \equiv n^{1/2}\widehat{\theta}^*/s(\widehat{\theta}^*)$ B times. Let $q_n^*(1-\alpha)$ denote the $100(1-\alpha)\%$ quantile of the empirical distribution of T_n^*. Comment on the properties of this test. (Assume the data are iid, $\widehat{\theta}$ is an asymptotically normal estimator of θ with a limiting standard deviation that is consistently estimated by $s(\widehat{\theta})$).

(ii) We are given data $z_1, ..., z_n$. Describe how to use subsampling to test a certain hypothesis H_0 against an alternative H_1 based on a certain test statistic $T_n = T_n(z_1, ..., z_n)$. Mention some of the key assumptions needed for subsampling to work and give the intuition behind those assumptions. What is the key difference between subsampling and the (nonparametric) bootstrap?