UCLA Department of Economics

First Year Core Examination in
Quantitative Methods

Fall 2008

This is a 4 hour closed book/closed notes exam.

Answer ALL questions in Parts I, II, and III-
Use a separate answer book for each part.

Calculators and other electronic devices are not allowed.

GOOD LUCK!
Quantitative Methods Comprehensive Examination

Part I (based on Ec203A)

Question 1:

Let \(X_1, \ldots, X_n\) be i.i.d. random variables, each distributed \(U(\theta_1, \theta_2)\), where \(\theta_1 < \theta_2\). In other words, for each \(i\), the density, \(f_{X_i}\), of \(X_i\) is

\[
f_{X_i}(x) = \begin{cases} 
\frac{1}{\theta_2 - \theta_1} & \theta_1 \leq x \leq \theta_2 \\
0 & \text{otherwise}
\end{cases}
\]

a. Show that the mean, \(\mu\), and variance, \(\sigma^2\), of \(X_i\), are given by \(\mu = (\theta_1 + \theta_2) / 2\) and \(\sigma^2 = (\theta_2 - \theta_1)^2 / 12\).

b. Provide consistent estimators for \(\mu\) and \(\sigma^2\). Prove that they are consistent.

c. Use your estimators for \(\mu\) and \(\sigma^2\) to derive consistent estimators, \(\hat{\theta}_1\) and \(\hat{\theta}_2\), for \(\theta_1\) and \(\theta_2\). Prove that your estimators are consistent.

d. Let

\[
\hat{\beta} = \sqrt{3n} \left( \frac{\hat{\theta}_2 + \hat{\theta}_1}{\hat{\theta}_2 - \hat{\theta}_1} \right)
\]

where \(\hat{\theta}_1\) and \(\hat{\theta}_2\) are the estimators that you derived in (c). Suppose, in this subquestion only, that \(E(X_1) = 0\). Derive the limiting distribution of \(\hat{\beta}\). Justify your answer.

e. Develop a test for the hypothesis \(H_0 : \theta_1 + \theta_2 = 5\) versus the alternative \(H_1 : \theta_1 + \theta_2 < 5\). Justify your steps.

f. Assume now that \(E(X_1) > 0\). Develop an approximate 95\% confidence interval for the value of the parameter \(\pi = \sqrt{(\theta_1 + \theta_2)}\).
Question 2:

Answer TRUE, FALSE, or UNCERTAIN, and justify.

1. Suppose that $X$ and $Y$ are independent random variables. Let $Z = g(X, Y)$ where $g$ is continuous function. Then, $Z$ and $X$ are independently distributed.

2. Suppose that $X$ and $Y$ are random variables such that for some function $h$, $Y = h(X)$. If $Y$ is continuous, $X$ may be either discrete or continuous, but, if $Y$ is discrete, $X$ must be discrete.

3. Suppose that the distribution of $X$, conditional on $Z$, is normal with mean $2Z$ and variance $\sigma^2$, and that $Z$ is normal with mean 0 and variance $\omega^2$. Then, the unconditional distribution of $X$ is normal with mean zero and variance $\sigma^2 + \omega^2$.

4. Suppose that the random vector $(X, Z)$ and the random variable $Y$ are independently distributed. Then, for all $x, y, z$, the conditional joint density of $(X, Y)$ given $Z$ and the conditional densities of $X$ given $Z$ and $Y$ given $Z$, satisfy

$$f_{X,Y|Z=z}(x, y) = f_{X|Z=z}(x) \cdot f_{Y|Z=z}(y)$$
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Part II (based on Ec203B)

PROBLEM 1:
TRUE, FALSE. EXPLAIN

(a) Suppose that the true model is \( Y_i = X_i \beta + \varepsilon_i \) where \( X_i \) is a scalar explanatory variable independent of \( \varepsilon_i \). Instead you run the OLS regression \( X_i = \gamma_i \delta + u_i \) and use \( 1/\hat\delta \) as an estimator of \( \beta \). The proposed estimator is consistent for \( \beta \).

(b) The \( R^2 \) from an OLS regression is the square of the simple correlation between the regressand and the predicted value.

PROBLEM 2:
Suppose that \( y_i = x_i \beta + \varepsilon_i \), where \( \varepsilon_i = \exp(z_i \delta) v_i \) and \( x_i \) is a sequence of i.i.d. random vectors, \( v_i \) is a sequence of i.i.d. random variables independent of \( x_i \) with \( E(v_i) = 0 \) and \( V(v_i) = 1 \), and \( z_i \) is a sequence of i.i.d. random vectors independently distributed of \( (x_i, v_i) \). Assume that the \( k \)-dimensional random vector \( x_i \) does not include a constant.

(a) Prove the consistency of the OLS estimator of \( \beta \) and derive the asymptotic distribution of \( \sqrt{n} \left( \hat\beta_{OLS} - \beta \right) \).

(b) Suggest a course of action for obtaining the "best" estimate for \( \beta \) under the assumptions of the model. In what sense is it best?

PROBLEM 3:
Suppose you have \( n \) independent observations \( \{(y_i, x_i)\}_{i=1}^n \), where \( y_i \) and \( x_i \) are scalar random variables. The density of \( y_i \) conditional on \( x_i \) is Gamma:

\[
\frac{(\beta + x_i)^{-\rho}}{\Gamma(\rho)} y_i^{\rho-1} e^{-y_i/(\beta + x_i)}
\]

Recall that for a Gamma distribution with density

\[
\frac{\lambda^\rho}{\Gamma(\rho)} y^{\rho-1} e^{-\lambda y}
\]

the mean is \( \rho/\lambda \) and the variance is \( \rho/\lambda^2 \). Propose three estimators for \( \beta \) and \( \rho \), and discuss their asymptotic properties.
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Part III (based on Ec203C)

1. Consider the Seemingly Unrelated Regression (SUR) model

\[ y_{ij} = x'_{ij}\beta_j + u_{ij} \quad (i = 1, \ldots, n; j = 1, \ldots, J) \]

(a) State the conditions for \( u_{ij} \) that would allow one to estimate the parameter vectors \( \beta_j, j = 1, \ldots, J \), from \( J \) separate least-squares (LS) regressions.

(b) Suppose now that for \( u_i = (u_{i1}, \ldots, u_{iJ})' \) we have

\[ u_i|x_{1i}, x_{2i}, x_{3i} \sim \text{i.i.d.} \, (0, \Sigma). \]

How would you obtain a feasible generalized least-squares (FGLS) estimator for \( \beta_1, \ldots, \beta_J \). Justify your answers at each and every stage.

(c) Suppose that it is given that \( \beta_2 = \beta_3 = \beta \). How would you get efficient estimates for \( \beta \) and \( \beta_1 \)? Justify your answers at each and every stage.
2. You are presented with a moment equations given by

\[ \varphi(w, \beta), \]

where \( \beta \) is a \( K \times 1 \) vector of unknown parameters, and with data \( w_i \), for \( i = 1, \ldots, n \), where \( w_i \) is a \( J \times 1 \) vector of data. Also, \( \varphi(w, \beta) \) is an \( M \times 1 \) vector-valued function. The true parameter vector is \( \beta_0 \), and our goal is to estimate \( \beta_0 \). Suppose that it has already been established that

\[ E[\varphi(w, \beta_0)] = E[\varphi(w, \beta)]|_{\beta=\beta_0} = 0. \]

(a) What are the minimal conditions required by the data and \( \varphi(w, \beta) \) that would allow one to estimate \( \beta_0 \). Please justify all statements made.

(b) Suppose now that one suggested a different \( M \times 1 \) moment vector-valued function, say \( \psi(w, \beta) \) that satisfies

\[ E[\psi(w, \beta_0)] = 0, \]

with \( M > K \). Describe in detail how to obtain the optimal generalize method of moments (GMM) estimator based on this latter function. Provide detailed justification.

(c) Suppose now that you are told that the model is given by

\[ y_i = h(x_i, \beta_0) + \epsilon_i. \]

What restrictions on \( \epsilon_i \) would allow you to estimate \( \beta_0 \)? Please justify your answer.

(d) Suppose the restrictions in (c) are satisfied. Suggest a moment function \( \psi(w, \beta_0) \) as in (b), and justify your answer.