Fall 2005 UCLA Department of Economics
Written Qualifying Examination in Quantitative Methods

Instructions:
Answer **ALL questions** in Parts I, II, and III
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

First Year Comprehensive Exam in Econometrics
Part I

1. (10 pt.) Consider the linear regression model

$$y = X\beta + \epsilon$$

where y, X, β, ϵ are $n \times 1$, $n \times k$, $k \times 1$, $n \times 1$ matrices, and $\epsilon \sim N(0, \sigma^2 I_n)$. We will assume that X is nonstochastic with full column rank. Let

$$\widehat{\beta} = (X'X)^{-1} X'y = \beta + (X'X)^{-1} X'\epsilon$$

denote the OLS estimator, and let

$$e = y - X\widehat{\beta}$$

denote the residual vector. Let c denote an arbitrary nonstochastic $k \times 1$ vector.

   (a) (2 pt.) Show that
   $$\widehat{\beta} - \beta \sim N\left(0, \sigma^2 (X'X)^{-1}\right)$$

   (b) (3 pt.) Show that $\widehat{\beta}$ and $e'e$ are independent of each other.

   (c) (2 pt.) Show that
   $$\frac{e'e}{\sigma^2} = \left(\frac{\epsilon}{\sigma}\right)' P \left(\frac{\epsilon}{\sigma}\right) \sim \chi^2 (n-k)$$

   (d) (3 pt.) Show that
   $$c'\widehat{\beta} \pm t_{.975}(n-1) \sqrt{s^2 c' (X'X)^{-1} c}$$
   where
   $$s^2 = \frac{e'e}{n-k}$$
   is a valid 95% confidence interval for $c'\beta$.

2. (10 pt.) Let Y_n be a statistic such that $\lim_{n\to\infty} E[Y_n] = \theta$ and $\lim_{n\to\infty} \operatorname{Var}(Y_n) = 0$. Prove that Y_n is a consistent estimator of θ. Hint: According to the Markov Inequality, we have

$$\Pr[|X| \geq c] \leq \frac{E\left[|X|^k\right]}{c^k}$$

for a random variable X and a nonstochastic constant c.

3. (10 pt.) Prove that, if $X_1, \ldots, X_n$ are independent Poisson random variables with means μ_i, then $\sum_{i=1}^n X_i$ is Poisson with mean $\sum_{i=1}^n \mu_i$. Hint: The MGF $M(t)$ of Poisson with mean m is equal to $\exp[m(e^t - 1)]$.

4. (10 pt.) Show that a sequence of random variables Y_n converges in probability to a (nonstochastic) constant c if and only if it converges in distribution to a limiting distribution degenerate at c.

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PART II

Problem 1: Consider a sample of n i.i.d. observations on (Y_t, X_t) from the model

$$Y_t = X_t \beta + \varepsilon_t \qquad t = 1, ..., n,$$

where ε_t is independent of X_t for all t.

(a) Provide conditions for the OLS estimator of β to be consistent.

(b) Derive the asymptotic distribution of the OLS estimator and provide a consistent estimator of its asymptotic covariance matrix.

(c) Describe how you would test the hypothesis that $\gamma(\beta) = 0$ for some known $p-$dimensional function γ with matrix of first derivatives Γ of full row-rank p.

Make all necessary moment assumptions to prove your claims. Make sure to prove your claims and state all necessary theorems.

Problem 2: Define the tobit model. Provide primitive conditions for consistency and asymptotic normality of the ML estimator and derive its asymptotic distribution.

First Year Comprehensive Exam in Econometrics
Part III

Question 1) True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

(a) In a linear regression model with iid data, HAC estimation is consistent, only if the bandwidth S_T grows to infinity when $T \to \infty$.

(b) With an MSE loss function, the optimal linear forecast of Y_t based on $(Y_{t-1}, ..., Y_1)$ does not depend on third and higher moments of the data *only if* the data are Gaussian.

(c) In a linear regression model with omitted regressors, OLS estimation is inconsistent.

(d) If the errors in a linear regression have a Cauchy distribution then the OLS estimator is better than the LAD estimator in terms of asymptotic relative efficiency.

(e) The linear difference equation $y_t = \alpha y_{t-2} + u_t$ where $u_t \sim iid\ N(0, \sigma^2)$ has a causal stationary solution if $|\alpha| < 1$. (Note, y_t is not an AR(1) here.)

Question 2)

(a) Is there a stationary solution y_t to the scalar difference equation $y_t = y_{t-1} + \varepsilon_t$ (where ε_t is white noise with zero mean and variance $\sigma^2 > 0$; Note: we do not assume $y_0 \equiv 0$). If yes, provide a solution, if no, prove your claim.

(b) If an AR(1) process is observed every other time period (i.e. instead of $y_1, y_2, y_3, \ldots$ you observe $y_1, y_3, y_5, \ldots$), is the observed process still AR(1)? If yes, express the parameters of the new process in terms of those of the original process, if no, prove your claim.

Question 3) Consider the linear simultaneous equations model

$$\begin{aligned} Y &= X\beta + \varepsilon, \\ X &= Z\Pi + v, \end{aligned}$$

where Y is $n \times 1$, X is $n \times d$. The observations are i.i.d. and you can assume conditional homoskedasticity. The sample size n is larger than d. Interest focuses on estimation of β.

Consider 2SLS estimators $\hat{\beta}_i$ $(i = 1, 2)$ based on instruments Z_i of dimensions $n \times k_i$, where $k_1 < k_2$ and Z_1 consists of the first k_1 columns of Z_2. Verify that the asymptotic variance of $\hat{\beta}_2$ is smaller than the one of $\hat{\beta}_1$ (in the positive definite sense).

Question 4) You want to test $H_0 : \theta = 0$ against $H_1 : \theta > 0$. The test for H_0 is to reject if $T_n = n^{1/2}\widehat{\theta}/s(\widehat{\theta}) > c$, where c is picked so that the type I error is α (the data are iid and as usually $\widehat{\theta}$ is a root$-n$ consistent estimator of θ and $s(\widehat{\theta})$ is a consistent estimator of the standard deviation). You do this as follows. Using the non-parametric bootstrap, you generate B bootstrap samples, calculate the estimates $\widehat{\theta}^*$ on these samples and then calculate $T_n^* := n^{1/2}\widehat{\theta}^*/s(\widehat{\theta}^*)$ B times. Let $q_n^*(1-\alpha)$ denote the $100(1-\alpha)\%$ quantile of the empirical distribution of T_n^*. You reject H_0 iff $T_n > q_n^*(1-\alpha)$. Discuss the power properties of this test.