Fall 2004 UCLA Department of Economics
Written Qualifying Examination in Quantitative Methods

Instructions:
Answer **ALL questions** in Parts I, II, and III
Use a separate answer book for each Part.
You have four hours to complete the exam.
Calculators and other electronic devices are not allowed.

1 Part I

Question 1 (10 pt): Let $X_1, \dots, X_5$ be i.i.d. with common PDF equal to $f(x;\theta) = \theta^x (1-\theta)^{1-x}$, $x = 0, 1$, zero elsewhere. We test $H_0 : \theta = \frac{1}{2}$ against $H_1 : \theta < \frac{1}{2}$ by rejecting H_0 if $\overline{X} = \frac{1}{5}(X_1 + \cdots + X_5)$ is observed less than or equal to a constant c.

1. (5 pt.) Find the significance level when $c = 1$.
2. (5 pt.) Show that this is a uniformly most powerful test under the significance level derived in (a).

Question 2 (10 pt): Suppose that U_i $i = 1, \dots, 64$, are i.i.d. $N(\mu, 1)$. Let $\overline{U} \equiv \frac{1}{64}\sum_{i=1}^{64} U_i$. What is the distribution of $\overline{U}$? If it happened that $\overline{U} = 3$, what would be your 95% confidence interval for μ?

Question 3 (10 pt): Let $X_1, \dots, X_n$ be i.i.d. $N(\theta, \sigma^2)$. Suppose that σ^2 is **known**. Show that $\overline{X} = n^{-1}\sum_{i=1}^{n} X_i$ has the minimal variance among all unbiased estimators. Justify your answer by explicitly invoking some theorem(s).

Question 4 (10 pt): Suppose that the moment generating function $E[\exp(t \cdot X)]$ of a random variable X is equal to $e^{\mu(e^t - 1)}$ for some $\mu > 0$. Show that $\text{Var}(X) = E[X]$.

2 Part II

Question 1:

True or False. Prove your claims.

(a) Suppose that you want to test a set of p linear restrictions on β in the model

$$Y = X\beta + \varepsilon$$

where $E(\varepsilon | X) = 0$. You may use the statistic

$$n \cdot \frac{SSR_R - SSR_U}{SSR_U}$$

which converges to a $\chi^2(p)$ as the sample size increases to infinity. Here SSR_R and SSR_U denote the sum of squared residuals from the restricted and unrestricted least squares regressions.

(b) Consider the partitioned regression model

$$Y = X_1\beta_1 + X_2\beta_2 + \varepsilon$$

Let $\tilde{Y}$ and $\tilde{X}_2$ be the residuals from the auxiliary regressions of Y and X_2 on X_1, respectively. The residual vector $\tilde{\varepsilon}$ from the regression of $\tilde{Y}$ on $\tilde{X}_2$ are equal to the residual vector $\hat{\varepsilon}$ from the original regression of Y on both X_1 and X_2.

Question 2:

A person decides to migrate depending on whether the present value of his/her lifetime utility at the present location, assumed to be determined by:

$$u_{i,p} = X_{i,p}\beta_p + \varepsilon_{i,p} \tag{1}$$

is less or equal to the present value of his/her lifetime utility at the migration location, assumed to be determined by:

$$u_{i,m} = X_{i,m}\beta_m + \varepsilon_{i,m} \tag{2}$$

minus the migration cost, assumed to be determined by:

$$C_i = Z_i\gamma + u_i \tag{3}$$

In equations (1)-(2) the subscripts p, m denote present and migration location. Variables in $X_{i,p}$ and $X_{i,m}$ would include, for example, the person's education, experience, age, race, gender, and local unemployment rates and average wages in different sectors. Variables in Z_i would include, for example, whether the individual is self-employed and whether he/she has recently changed industry of employment. $\varepsilon_{i,p}$, $\varepsilon_{i,m}$, and u_i are unobserved error terms that are jointly normally distributed with zero means and a positive definite covariance matrix

$$\Sigma = \begin{bmatrix} \sigma_{pp} & \sigma_{pm} & \sigma_{pu} \\ & \sigma_{mm} & \sigma_{mu} \\ & & \sigma_{uu} \end{bmatrix}$$

Suppose that we can observe whether an individual has decided to migrate or not, and $X_{i,p}$, $X_{i,m}$, and Z_i which are assumed to be independent of all error terms.

(a) Construct the econometric model for the migration decision for an individual i.

(b) Construct the log-likelihood for a sample of n i.i.d. observations of the migration decision of individuals.

(c) Describe as precisely as you can the maximum likelihood of the unknown parameters in equations (1)-(3). Can all of the coefficients in equations (1)-(3) be consistently estimated?

(d) Derive are the asymptotic properties of the estimators of part (c) above.

3 Part III

Question 1) True/Questionable/False? (No points are given for just stating true/questionable or false. The explanation is what counts.)

(i) When instruments are weak, a large value of the Wald statistic is not necessarily strong evidence against the null hypothesis.

(ii) When forecasting an MA(1) process based on its past observations, the mean squared error of the forecast goes to zero with the sample size growing to infinity.

(iii) In general, in HAC estimation, the bandwidth S_T has to grow to infinity as the sample size T goes to infinity to guarantee that the variance of the HAC estimator converges to zero.

(iv) In the standard linear regression model with errors distributed as t with v degrees of freedom, OLS is more efficient than LAD.

(v) When testing overidentifying restrictions, a large value of the J-test is not necessarily evidence against the null hypothesis of instrument exogeneity due to the inconsistency of the test against certain alternatives.

Question 2) For a covariance stationary process Y_t derive a formula [in terms of $\mu := E(Y_t)$, γ_0, γ_1, and γ_2, where γ_k denotes the covariance of Y_t at lag k] for the linear projection of Y_{t+1} on a constant and Y_{t-1}. Using that result, calculate this linear projection if Y_t is an AR(1) process given by $Y_t = c + \phi Y_{t-1} + \varepsilon_t$.

Question 3) Consider a model given by the system of equations

$$\begin{aligned} y_i &= Y_i'\beta + W_i'\gamma + u_i = X_i'\delta + u_i \text{ and} \\ Y_i &= Z_i'\pi + V_i, \end{aligned}$$

where the observations are i.i.d. over $i = 1, ..., n$ and $(u_i, V_i')' \sim N(0, \Lambda)$. Let $\widehat{\delta} := (\widehat{X}'\widehat{X})^{-1}\widehat{X}'y$ be the 2SLS estimator of δ, where $\widehat{X} := P_Z X$, $X := [X_1, ..., X_n]'$, $Z := [Z_1, ..., Z_n]'$, $P_Z := Z(Z'Z)^{-1}Z'$, and $y := [y_1, ..., y_n]'$.

(i) Give an example of such a model in Economics. What are the variables Y_i, W_i, and Z_i in your example?

(ii) From first principles, show that $\widehat{\delta}$ is consistent and asymptotically normal. State any (additional) assumptions that are needed.