Fall 2003

Department of Economics
UCLA

Quantitative Methods Comprehensive Examination

This is a four hour closed-book examination. There are three parts in this exam. Please answer **ALL** parts of the exam. Use separate exam book for each of the three sections.

Calculators, or any other electronic devices, are not allowed.

Part I.

1. Suppose that a random variable X has the PDF $f_X(x)$ such that
$$f_X(x) = \frac{\exp(-\theta)\,\theta^x}{x!} \qquad x = 0, 1, 2, \ldots$$
Compute the Fisher Information for θ.

2. Let Z_n denote some sequence of random variables such that $\sqrt{n}(Z_n - 1) \xrightarrow{d} N(0, 4)$. Derive the asymptotic distribution of $\sqrt{n}(Z_n^2 - 1)$.

3. Suppose that $X_i \sim N(\mu_i, \sigma_i^2)$ are independent of each other. Show that $\sum_{i=1}^n a_i X_i \sim N\left(\sum_{i=1}^n a_i\mu_i, \sum_{i=1}^n a_i^2\sigma_i^2\right)$. Hint: The moment generating function of $N(\mu, \sigma^2)$ is equal to $\exp\left(\mu t + \frac{\sigma^2}{2}t^2\right)$.

4. Suppose that $X_1, X_2, \ldots, X_{100}$ are i.i.d. with PDF equal to
$$\Pr[X_i = y] = \frac{\theta^y e^{-\theta}}{y!}, \quad y = 0, 1, 2, \ldots$$
   (a) Show that
   $$E\left[X_i^2\right] = \theta + \theta^2$$
   Hint: Recall that $E[X_i] = \mathrm{Var}(X_i) = \theta$.

   (b) You found that
   $$\sum_{i=1}^{100} X_i^2 = 600$$
   Compute the Method of Moments estimator for θ.

Part II.

1. Suppose that we have data z distributed according to a density $f(z, \theta)$ that is characterized by an unknown parameter vector $\theta \in \Re^k$.

   (a) Write down the form of Wald's test statistic for testing a general nonlinear hypothesis of the form $\gamma(\theta) = 0$, where $\gamma()$ is a $q-$valued function with a Jacobian of full row rank q and derive the asymptotic distribution of the test statistic stating appropriate theorems and assumptions that you use in your proof.

(b) Derive the form of the Wald statistic in the case of the generalized normal linear regression model $y = X\beta_0 + \varepsilon$ where $\varepsilon \sim N(0, \Omega)$ for testing a set of q linearly independent restrictions of the form $\Gamma\beta_0 = \gamma$.

2. Suppose that you have n independent observations $\{y_i\}_{i=1}^n$ distributed according to the density

$$f(y_i) = \left(\frac{1}{x_i\beta}\right) \exp\left(-\frac{y_i}{x_i\beta}\right) \qquad y_i > 0$$

where x_i is a $k-$dimensional vector of constants and β is an unknown vector of parameters.

(a) Verify that $E(y_i) = x_i\beta$ and $V(y_i) = (x_i\beta)^2$.

(b) Derive the Maximum Likelihood estimator of β.

(c) Derive the Generalized Least Squares estimator of β.

(d) Derive the asymptotic distribution of the estimators of parts (b) and (c).

Part III.

1. Consider the two-equation linear model given by

$$\begin{aligned} y_{1i} &= x_{1i}'\beta_1 + u_{1i}, \\ y_{2i} &= x_{2i}'\beta_2 + u_{2i}, \end{aligned}$$

where x_{1i} and x_{2i} are $K_1 \times 1$ and $K_2 \times 1$ vectors of regressors, respectively, β_1 and β_2 are the corresponding unknown vectors of parameters. Assume that $K_2 > K_1$. Let $u_i = (u_{1i}, u_{2i})'$, and assume that

$$u_i | x_{1i}, x_{2i} \sim \text{i.i.d. } (0, \Sigma_u).$$

(a) Describe how to obtain the most efficient estimates for β_1 and β_2.

(b) Provide a consistent estimate for Σ_u.

(c) Suppose that $\text{Cov}(u_{1i}, u_{2i}) = \sigma_{12}$, where σ_{12} is a known parameter (i.e., it is a number that the econometrician knows). How would it change the answer in (1)?

(d) Suppose that all the variables in x_{1i} are also in x_{2i}, that is, $x_{1i} \subset x_{2i}$ for all $i = 1, ..., n$. How would it change the answer in (1) with respect to the parameter vector β_1 and with respect to the parameter vector β_2?

(e) Suppose now that

$$\begin{aligned} y_{1i} &= x_{1i}'\beta_1 + \gamma y_{3i} + u_{1i}, \\ y_{2i} &= x_{2i}'\beta_2 + u_{2i}. \end{aligned}$$

Provide a methods for consistently estimating β_1 and β_2.

2. Consider the binary discrete choice model given by

$$\Pr(y_i = 0) = \frac{\exp(x_i'\gamma)}{1 + \exp(x_i'\gamma)},$$

for $i = 1, ..., n$.

(a) Provide the MLE for γ, say $\widehat{\gamma}_n$.

(b) Show that the MLE estimator can be viewed as a Method of Moments (GMM) estimator.

(c) Compute the exact asymptotic covariance for $\widehat{\gamma}_n$ and provide a consistent estimate for the asymptotic covariance. Justify your answer.

(d) Consider the weighted estimator, say $\widehat{\gamma}_n^W$, obtained by

$$\min_{\gamma} \sum_{i=1}^{n} \frac{(y_i - \Pr(y_i|x_i))^2}{\Pr(y_i|x_i)(1 - \Pr(y_i|x_i))}.$$

Is $\widehat{\gamma}_n^W$ consitent estimator for γ? Justify your answer.

(e) Assume now that the estimator obtained in (d) is consistent estimator for γ. Would you prefer that estimator on the one obtained in (a)? Justify your answer.