UCLA Economics

Fall 2002 Quantitative Methods Comprehensive Examination

There are three sections with two questions in each section. Please answer all the questions; use a separate blue book for each of the three sections. For your information, the .90 and 0.95 quantiles of the Chi-squared distribution with one degree of freedom are 2.78 and 3.84.

Part I.

1. Let X be a binary variable with $Pr(X=1)=2/3$ and $Pr(X=0)=1/3$. Conditional on $X=x$, the random variable Y has an Poisson distribution with arrival rate $\theta + x$.

    1. What is the mean of Y.
    2. What is the probability that $X=1$ given $Y=0$.
    3. Suppose that $(X_1, Y_1), \ldots, (X_N, Y_N)$ are a random sample from this distribution. What is the maximum likelihood estimator for θ?
    4. What is the large sample variance for the maximum likelihood estimator?
    5. Construct a moment estimator for θ based on the the expected value of Y. What is the variance of this estimator? How does it compare to the variance of the maximum likelihood estimator?

    Note: the probability mass function for a Poisson random variable Z with parameter λ is, for nonnegative integer z:

    $$f_Z(z;\lambda) = \lambda^z \exp(-\lambda)/z!.$$

2. Let $X_1, X_2, \ldots, X_N$ be a random sample from the density

    $$f(x;\lambda) = \lambda^{-1}(1+x)^{-\left(\frac{\lambda+1}{\lambda}\right)}$$

    for $x > 0$ and $\lambda > 0$.

    1. Find the maximum likelihood estimator for λ.
    2. Find the maximum likelihood estimator for $1/\lambda$.
    3. Find the Cramer–Rao bound for unbiased estimators of λ.
    4. Is the maximum likelihood estimator for λ equal to the minimum variance unbiased estimator for λ? Is the maximum likelihood estimator for $1/\lambda$ equal to the minimum variance unbiased estimator for $1/\lambda$?
    5. Suppose the maximum likelihood estimator for λ is equal to 1, and the number of observations is 100. Test the hypothesis $\lambda = 0.9$ at the 5% level using a likelihood ratio test.
    6. Repeat the test using a Wald test.

Part II.

1. For the $k-$variate linear regression model,

$$y_i = x_i\beta + \varepsilon_i$$

where $E(x_i'\varepsilon_i) = 0$ show that the OLS estimator is consistent and asymptotically normal assuming that you have a random sample on (y_i, x_i) of size n. Provide a consistent estimator of its variance-covariance matrix. Is it unbiased? Make sure to state the theorems used in obtaining your results. Do any of your results change if in addition you know that ε_i is distributed normally conditional on x_i with mean zero and constant variance σ^2?

2. Consider the model

$$C_t = \alpha + \beta Y_t + \gamma C_{t-1} + \varepsilon_t \qquad |\gamma| < 1; \qquad t = 1, ..., T \qquad (1)$$

where C_t and Y_t are consumption and income, respectively, at time t. It is assumed that $\{\varepsilon_t\}$ is an i.i.d. zero-mean process with finite variance σ_ε^2, and $\{Y_t\}$ is an i.i.d. process with finite mean μ_Y and finite variance σ_y^2 which is independent of $\{\varepsilon_t\}$. In this model, the long-run marginal propensity to consume is defined as $\delta = \frac{\beta}{1-\gamma}$.

   1. Find the mean and variance of $\{C_t\}$.
   2. Does OLS on (1) produce consistent estimates of α, β, and γ? Are the OLS estimators unbiased? Justify your answer.
   3. Derive the (joint) asymptotic distribution of the OLS estimators of part (b), assuming that appropriate conditions hold so that you can apply a central limit theorem.
   4. Consider estimating δ by $\hat{\delta} = \frac{\hat{\beta}_{OLS}}{1-\hat{\gamma}_{OLS}}$. Is it unbiased, consistent, asymptotically normal? Justify your answer.
   5. Using data on consumption and income from 1963 to 1972, we estimated (1) by OLS which produced the following results:

	Coefficient	Standard Error	$t-$ratio
Constant	-7.69575	11.44	-0.672
Y_t	0.400147	0.06272	6.380
C_{t-1}	0.380728	0.09479	4.017

Estimated Covariance Matrix of Estimates

	Constant	Y_t	C_{t-1}
Constant	130.972		
Y_t	-0.43868	0.00393	
C_{t-1}	0.54959	-0.0058961	0.00898

Test the hypothesis that $\delta = 1$.

Part III.

1) Assume that $y_t = X_t\beta + \rho y_{t-1} + \varepsilon_t$ where $\varepsilon_t = \alpha_1\omega_t + \alpha_2\omega_{t-1}$. Furthermore, the ω's are independently and identically distributed and $E\{\omega_t|X_\tau\} = 0$ for all t and τ. However, the exact distribution of the ω's is unknown.

   a) Show that the OLS regression of y_t on X_t and y_{t-1} is an inconsistent estimator for β and ρ?

   b) Construct the most efficient, consistent estimator of β and ρ that you can.

2) Assume that $q_i = X_i\beta + \varepsilon_i$ determines the quantity of a particular security that the i^{th} individual desires to buy, where the error terms are normal random variables that are independent across individuals $i = 1, 2, \ldots, n$.

   a) In the first period, nobody is allowed to short the stock (i.e. you cannot purchase a negative quantity). Given data on sales for these n individuals, how would you estimate β? Be explicit.

   b) In the second period, agents can short the stock. Given data only from this second period on the same n individuals, how would you estimate β? Be explicit.

   c) How would you combine the data from the two periods to construct an efficient estimator of β? Be explicit and note that although the error terms are independent across individuals, they may correlate through time for a given individual.

   d) How would you test the hypothesis that peoples' behavior (as represented by β) remains the same across the two periods? Again, be explicit.