UCLA, Department of Economics

MICRO COMPREHENSIVE EXAMINATION
SPRING 1999

Time allowed: four (4) hours. You must use a separate blue book for each question. Answer five (5) out of the six (6) questions.

1. Aggregation and Choice over time

(a) Consumer h, $h = 1,\dots,H$ has a utility function $u(c^h) = \sum_{t=1}^{T} \alpha_t \ln c_t^h$. His income is I^h and he faces a price vector $p = (p_1,\dots,p_T)$. Show that aggregate demand can be represented by a single consumer with the same preferences and aggregate income $I = \sum_{i=1}^{H} I^h$

(b) Suppose that $\alpha_t = \delta^{t-1}$, where $\delta < 1$ so the consumption vector can be interpreted as consumption of a single commodity over time. The aggregate endowment vector is $(\omega,0,\dots,0)$. That is, the only endowment is in the initial period. Goods can be stored. However, for each period that a good is stored, only the fraction $\theta < 1$ is available the next period. The remainder is spoiled. Appealing to part (a) or otherwise, solve for the optimal consumption of the representative individual and also for the Walrasian equilibrium prices when T=2 (two periods.)

(c) What is the equilibrium interest rate? How would it change if the discount factor were to rise? How would the interest rate change if the parameter θ were to rise?

Henceforth assume that the aggregate endowment is the same in each period.

(d) What are the new equilibrium prices and interest rate?

(e) Suppose that there is also a constant returns to scale technology. Input of x_t units at time t yields $y_{t+1} = (1+\alpha)x_t$ units of output at time $t+1$, where $\alpha > 0$. What is the new equilibrium interest rate?

2. Short-takes

In each case indicate whether "true", "false" or "maybe" and explain carefully.

(a) Consider a complete state claims equilibrium of an exchange economy with aggregate endowment $(\omega_1,\dots,\omega_S)$ where $\omega_1 > \dots > \omega_S$.. The probability of state s is π_s If some individuals are risk neutral, the state claims prices will be proportional to the probability vector π.

(b) In the same economy, if all individuals are risk averse, $\frac{p_s}{p_{s+1}} < \frac{\pi_s}{\pi_{s+1}}$..

(c) A serious limitation of the exchange model with complete state claims is that the first welfare theorem only holds if individuals have the same beliefs.

(d) In a 2×2 constant returns to scale world, suppose country A and B are small so that they take the equilibrium international price ratio to be constant. Country A is a more efficient producer. Given the same inputs it can produce twice as much of either commodity. In such a world the input prices will be twice a high in country A as in country B.

(e) Suppose instead that to achieve the same output requires the same level of capital but twice as much labor. In this case one might expect the rental rate on capital to be the same in the two countries.

3. Tester's Curse

The Food and Drug Administration of Ruritania (FDA) has responsibility for approving or disapproving new drugs. It seeks to approve drugs that are safe and disapprove drugs that are unsafe. Unfortunately, whether a drug is safe or unsafe can only be determined with certainty after it has been in general use for one year. Ruritanian law therefore mandates that the FDA obtain independent tests from 2 laboratories, accept the drug if **both** laboratories report that the drug passed the test, reject the drug otherwise. Unfortunately, these tests are not very good: if the drug is safe, it will pass each individual test 60% of the time; if the drug is unsafe, it will pass each individual test 30% of the time, and results of independent tests will be independent. Of all drugs submitted, 50% are safe, 50% are unsafe.

The FDA pays its two independent laboratories according to the following scheme:

- If the drug is accepted and proves safe: 1000
- If the drug is accepted and proves unsafe: 0
- If the drug is rejected: 500

The heads of the laboratories have Ph.D.'s in Economics from UCLA, so understand strategic optimization and care only about monetary payoffs.

a) Formulate this situation as a Bayesian game. Be sure to identify information and strategy.
b) Is it a Bayesian Nash equilibrium for both laboratories to report honestly?
c) Find a Bayesian Nash equilibrium in pure strategies?
d) Is there a Bayesian Nash equilibrium in which some drugs are accepted and some are rejected?

4. Repeated Games

ROW and COL play the game below several times, discounting future payoffs according to the discount factor $\rho = ½$. **All questions below refer to equilibria in pure strategies.**

a) The game is played 2 times: what is the highest payoff ROW can obtain in any Nash equilibrium (in pure strategies)?

b) The game is played 2 times: what is the highest payoff ROW can obtain in any subgame perfect equilibrium (in pure strategies)?

c) The game is played 3 times: what is the highest payoff ROW can obtain in any subgame perfect equilibrium (in pure strategies)?

d) The game is 4 times: what is the highest payoff ROW can obtain in any subgame perfect equilibrium (in pure strategies)?

	W	X	Y	Z
A	(64,0)	(0,0)	(0,5)	(0,0)
B	(0,0)	(32,0)	(0,0)	(0,5)
C	(80,0)	(0,0)	(16,8)	(0,0)
D	(0,0)	(0,0)	(0,0)	(8,16)

5. Edgeworth Variations

Consider an exchange economy with two consumers and two commodities. Both consumers have consumption set $X_i = [0, 10]^2$. Consumer 1 has utility function $U(x_1, x_2) = \sqrt{x_1 x_2}$ and endowment $w_1 = (4, 0)$. Consumer 2 has utility function

$$U(x_1, x_2) = \max\{\min\{2x_1, x_2\}, \min\{x_1, 2x_2\}\}$$

and endowment $w_2 = (0, 4)$. Normalize prices such that $p_1 + p_2 = 1$.

(a) Compute the demand correspondence of consumer 1, being careful to restrict consumption choices to the consumption set. Graph the image of this correspondence in $\mathbf{R}^2_+$. For which values of $p_1 \in [0, 1]$ is the demand correspondence upperhemicontinuous? Lowerhemicontinuous? Continuous?

(b) Compute the demand correspondence for consumer 2, again being careful to restrict consumption choices to the consumption set. Graph the image of this correspondence in $\mathbf{R}^2_+$. For which values of $p_1 \in [0, 1]$ is the demand correspondence upperhemicontinuous? Lowerhemicontinuous? Continuous.

(c) Construct the Edgeworth box diagram for this economy. Find the Walrasian equilibria, if there are any.

(d) Suppose instead that consumer 2 has utility function

$$U(x_1, x_2) = x_2$$

with everything else remaining the same. Describe the demand correspondence for this consumer. Find the Walrasian equilibria or quasi-equilibria for this case (and explain the difference between an equilibrium and a quasi-equilibrium).

6. Equilibrium on an Event Tree

Consider a production economy defined on a event tree with three dates: $t = 0, 1, 2$. For $t = 0, 1$ each node in the tree is followed by two branches, labeled U (up) and D (down), each occurring with probability 1/2. There is one physical commodity available at each of the date-events (t, a_t) in the tree. There are n consumers each with utility function

$$U_i(x_i) = \sum_{t=0}^{2} \sum_{a_t \in f_t} \delta^t \pi(a_t) \ln(x_i(t, a_t))$$

where $\delta \in (0, 1]$ is a discount factor, a_t is an event in the partition f_t of observable events at date t, $\pi(a_t)$ is the probability of event a_t, and $x_i(t, a_t)$ is the quantity consumed by consumer i at date-event (t, a_t). All consumers have endowment $w_i(0) = 7$ at date 0 and $w_i(t, a_t) = 0$ for all other date-events. At each non-terminal date-event there are two production technologies.

- Technology 1 uses 1 unit of the commodity at (t, a_t) to deliver 2 units of the commodity at date $t + 1$ if the intervening branch is a U.
- Technology 2 uses 1 unit of the commodity at (t, a_t) to deliver 2 units of the commodity at date $t + 1$ if the intervening branch is a D.

The economy can use both technologies simultaneously, with each subject to constant returns to scale. Assume $\delta = 1/2$ and normalize prices such that $p(0) = 1$.

(a) Find the Arrow-Debreu equilibrium for this economy including the Arrow-Debreu prices, the equilibrium allocation to each consumer, and the aggregate activity (production) vector. Decompose the aggregate activity vector into six activity vectors corresponding to technologies 1 and 2 operating at date 0 and (the two events) at date 1.

(b) Assume that contingent commodities are available only one date in advance. Normalizing spot prices to equal 1 at every date-event, find the Radner equilibrium for this economy (i.e., find the prices, allocation to each consumer, activity vector, and trades in the contingent commodity).

(c) Suppose now that technologies 1 and 2 represent production activities of the individual consumers: i.e., each consumer uses his or her available amount of the commodity at date t to produce the commodity at date $t + 1$ either in the event of a U (technology 1) or in the event of a D (technology 2). Furthermore, at any node a consumer is able to operate only technology 1 or technology 2, not both. Is there a Radner equilibrium if $n = 2$? If $n = 3$? With a continuum of consumers?