**UCLA Economics Dept.** **Ph.D. Qualifying Exam**
**Spring 1998**
**Microeconomics**

**Do 5 Questions; if you do more than 5 Questions, only the first 5 will be graded. Write each question in a separate Blue Book. Show the Question number and your identifying number on the front of the Blue Book.**

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## Part I

### #1: Life Cycle

An individual lives for 3 periods. His lifetime utility function is:

$$U(c) = v(c_1) + \delta v(c_2) + \delta^2 v(c_3)$$

where $\delta$ is a discount factor, $0 < \delta < 1$ and $v$ is increasing and concave. In period $t$ the individual earns a wage $w_t$. In each period he can save and earn an interest rate $r$, or borrow at the interest rate $m \geq r$. He begins in period 1 with initial capital $K_1 = 0$.

(a) Assume $w_1 = w_2 = w_3 = w$ [constant wages] and that $m > r$. Under what conditions will this individual save in period 1? Derive your result fully.

(b) Assume $w_1 = w_2 = w_3 = w$ [constant wages] and that $m > r$. Derive the possible life-cycle consumption and saving/borrowing patterns.

(c) Assume that $w_1 = w_2 = w$, $w_3 = 0$ [retirement in period 3], $m = r$ [borrowing and lending rates equal] and that $v(c_t) = \log c_t$ [natural logarithm]. Under what conditions (if any) will he borrow in period 1 and save in period 2?

## #2: Constant Returns to Scale Economy

Product 1 is produced according to the linear production function

$$x_1 = 2\ell_1 + 4k_1$$

Product 2 is produced according to the Leontieff [fixed coefficients] production function

$$x_2 = \min\{\frac{\ell}{2}, k_2\}$$

The aggregate supply of labor and capital is $(\bar{\ell}, \bar{k}) = (80, 60)$.

Suppose first that there is no international trade.

(a) Depict the Edgeworth Box in a neat figure with labor on the horizontal axis and capital on the vertical axis.

(b) Depict also the production possibility frontier. Assuming that both products are consumed, what are the possible equilibrium output price ratios? Explain carefully.

Henceforth suppose that this country can trade at fixed international prices

$$(p_1, p_2) = (\frac{1}{12}, 1)$$

(c) What will be the wage and rental rates in this economy?

(d) Determine the equilibrium wage-rental ratio for all possible levels of the aggregate labor supply $\bar{\ell}$.

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**Part II**

**# 3: Repeated Games**

$\Gamma$ is the following game:

| | L | C | R |
|---|---|---|---|
| U | 10, 10 | 4, 12 | 0, -20 |
| M | 12, 4 | 0, 0 | 0, -20 |
| D | -20, 0 | -20, 0 | -20, -20 |

The following questions concern the infinitely repeated game $\Gamma^{\infty}(\rho)$ in which $\Gamma$ is played in each period and both players discount future payoffs using the common discount factor $\rho$.

a) If $\rho = 1/5$: show that there is no subgame perfect equilibrium in which UL is played in every round (along the equilibrium path).

b) If $\rho = 1/5$: Find a Nash equilibrium in which UL is played in every round (along the equilibrium path) and verify carefully that your solution is a Nash equilibrium.

c) If $\rho = 4/5$: Find a subgame perfect equilibrium in which UL is played in every round (along the equilibrium path), and verify carefully that your solution is a subgame perfect equilibrium.

# # 4: Bayesian Games

ROW and COLUMN are playing a simultaneous move (normal form) game. ROW knows the payoffs, COLUMN is unsure. COLUMN assigns probability 1/3 that the payoffs are:

| | L | R |
|---|---|---|
| U | 2 | 2 |
| | -2 | 0 |
| D | 0 | 0 |
| | 2 | 0 |

COLUMN assigns probbability 2/3 that the payoffs are:

| | L | R |
|---|---|---|
| U | 0 | 2 |
| | 2 | 0 |
| D | 2 | 0 |
| | -2 | 0 |

[ROW knows COLUMN's probability assessments and ROW knows that COLUMN knows, etc.]

Find all the Bayesian Nash equilibria.

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## Part III

### #5: Production under Uncertainty

Consider a production economy with commodity space defined on an event tree with probability space $\Omega$ and time set $\{0, 1, \ldots, T\}$ where the horizon $T < \infty$. Let $f_t$ denote the partition of $\Omega$ into events $a_t \subset \Omega$ observed at date $t$.

There is a single consumer with utility function

$$U(x) = \sum_{t=0}^{T} \sum_{a_t \in f_t} \pi(a_t) \left[\frac{1}{2} \ln(x_1(t, a_t)) + \frac{1}{2} \ln(x_2(t, a_t))\right]$$

where $\pi(a_t)$ is the probability of event $a_t \in f_t$ and $x_j(t, a_t)$ is the quantity of commodity $j$ consumed at $(t, a_t)$. [Note: to simplify computation, we have assumed a discount factor $\delta = 1$.] Consumer endowment $w$ is given by

$$w(t, a_t) = (w_1(t, a_t), w_2(t, a_t)) = \begin{cases} (k, 0) & \text{if } t > 0 \\ (k, k) & \text{if } t = 0 \end{cases}$$

Each node $(t, a_t)$ of the tree for $t < T$ has three immediate successors labeled $a_{t+1}^L$, $a_{t+1}^M$, and $a_{t+1}^H$. At any date $t < T$, commodity 2 can be produced with a one-period lag using any non-negative multiple of the production vector

$$(y_1(t, a_t), y_2(t + 1, a_{t+1}^L), y_2(t + 1, a_{t+1}^M), y_2(t + 1, a_{t+1}^H)) = (-1, 1, 2, 3)$$

where $0 \leq t < T$. To simplify your answer, let $I(t, a_t)$ denote the input ("investment") in production at date $t$, and normalize prices so that $p \cdot w = 1$.

Find the Arrow-Debreu prices and allocation for this economy as functions of $k$ and $\pi(a_t)$.

# # 6: Exchange with Quasi-Linear Utility

Consider an exchange economy with three commodities and a continuum of consumers, $I = [0, 1]$ with Lebesgue measure. Assume that all consumers have the same endowment

$$w_i = (k, k, k) \quad \text{where} \quad k > 0$$

Normalize prices to sum to one. Assume that consumer $i \in I$ has utility function

$$U_i(x_i) = x_{i1} + 2i\sqrt{x_{i2}x_{i3}}$$

(a) Assuming an interior maximum, what are the first-order conditions for a utility maximum for consumer $i \in I$? Are these first-order conditions plus the budget constraint sufficient to determine the demand function?

(b) Find the demand function (correspondence) and the indirect utility function for consumer $i$.

(c) Find the Walrasian equilibrium allocation and equilibrium prices for this economy. Illustrate the equilibrium allocation for each of the three commodities in a diagram with $I$ on the horizontal axis and quantity on the vertical axis. Give a similar plot of the equilibrium utility allocation.