UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2014)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Classical Equilibrium Existence Theorem

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, \succeq_i, e_i\}_{i \in I})$ with free disposal technology, where $\mathbb{X}_i = \mathbb{R}^L_+$ and $\succeq_i$ is rational, continuous and strictly convex. Also assume that i's upper contour set $U_i(x'_i) = \{x_i \in X_i | x_i \succeq_i x'_i\}$ is bounded for every $x'_i \in X_i$ for any i (hence $\succeq_i$ is satiated, i.e. there exists $\widehat{x}_i \in X_i$ such that $\widehat{x}_i \succeq_i x_i$ for all $x_i \in X_i$). Answer the following questions.

(a) Define Walrasian equilibrium in this economy.

(b) Prove that Walrasian demand correspondence $x_i(p)$ can be defined on $\mathbb{R}^L_+/\{\mathbf{0}\}$ and it is a continuous function.

(c) Prove that there exists a Walrasian equilibrium in this economy. Do not use any Walrasian equilibrium existence theorem (but you can use a fixed point theorem).

2. Subjective Expected Utility

Let S be a set of finite states, X be a space of outcome, and Π be the space of simple lotteries on X (i.e. the set of finite support distributions on X). Let $f : S \to \Pi$ be an act and $\mathcal{F}$ be the set of all acts. Consider a decision maker who has a preference $\succeq$ on $\mathcal{F}$. Suppose that $\succeq$ can be represented by a function $U : \mathcal{F} \to \mathbb{R}$ ($f \succeq g$ if and only if $U(f) \geq U(g)$) that satisfies $U(\alpha f + (1-a)g) = \alpha U(f) + (1-\alpha)g$ for any $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$. Answer the following questions.

(a) When can you find such U to represent $\succeq$? Describe three axioms on $\succeq$ to guarantee the existence of such U (no proof is needed).

(b) Prove that there exists $u_s : X \to \mathbb{R}, s \in S$ such that for any $f, g \in \mathcal{F}$, the following holds:

$$f \succeq g \text{ if and only if } \sum_{s\in S}\left[\sum_{x\in \mathrm{supp}(f_s)} u_s(x) f_s(x)\right] \geq \sum_{s\in S}\left[\sum_{x\in \mathrm{supp}(g_s)} u_s(x) g_s(x)\right].$$

(c) Suppose that $\succeq$ satisfies the following axiom:

$(*)$ For any $p, q \in \Pi$,
if $(p, ..., p) \succeq (q, ..., q)$, then $(p, f_{-s}) \succeq (q, f_{-s})$ for any $f \in \mathcal{F}$ and $s \in S$,

where (p, f_{-s}) is the act that is obtained by replacing f_s with lottery $p \in \Pi$.

Prove that there exists a distribution μ on S and $u : X \to \mathbb{R}$ such that for any $f, g \in \mathcal{F}$, the following holds:

$$f \succeq g \text{ if and only if } \sum_{s\in S}\mu(s)\left[\sum_{x\in \mathrm{supp}(f_s)} u(x) f_s(x)\right] \geq \sum_{s\in S}\mu(s)\left[\sum_{x\in \mathrm{supp}(g_s)} u(x) g_s(x)\right].$$

3. Perfect Bayesian Equilibrium

Consider an environment with one Firm and one Consumer. The Firm produces a divisible good at 0 fixed cost and constant marginal cost $k = 1$. If the Consumer purchases x units of the good at a per-unit price of p then the firm makes profit

$$(p-1)x$$

and the Consumer experiences utility that depends on the state of nature: Good G or Bad B:

$$\begin{aligned} u_G &= 6x - x^2 - px \\ u_B(x) &= 2x - x^2 - px \end{aligned}$$

(In what follows, assume the money endowment of the Consumer is so large that the non-negativity constraint never binds.)

The true weather is Good or Bad with equal probability; this is common knowledge. The Firm learns the true weather; after learning the true weather the Firm offers the good at a price p. The Consumer does not learn the true weather but observes the offered price and buys as much or as little of the good as desired. The Firm seeks to maximize profit; the Consumer seeks to maximize (expected) utility.

This defines a Bayesian game between the Firm and the Consumer: the Firm observes the weather and offers a price, the Consumer chooses a quantity. We are interested in pure strategy equilibria only.

(a) Find the pooling Perfect Bayesian Nash equilibrium (i.e. a PBNE in which the the firm offers the same price in each state) that is best for the firm in the sense of yielding the firm the largest *ex ante* expected profit (the largest expected profit *before* the Firm learns the weather).

(b) Find a separating Perfect Bayesian Nash equilibrium (i.e. a PBNE in which the firm offers different prices in each state) in which the firm makes positive profits in both states. Is this separating PBNE unique?

(c) Show that the pooling PBNE equilibrium you found in (a) is better for the firm than *every* separating PBNE equilibrium .

4 Repeated Games

The stage game G below is played infinitely often; players use the discount factor $\delta \in (0,1)$. (We have called the infinitely repeated game $G^{\infty}(\delta)$.)

$G =$

	L	R
U	3,0	-1,-1
D	2,2	0,3

(a) Find a discount factor $\delta \in (0,1)$ and a subgame perfect equilibrium strategy profile σ for $G^{\infty}(\delta)$ in which, on the equilibrium path (i.e. when no deviations have occurred), (D,L) is played in every period.

(b) Find a discount factor $\delta \in (0,1)$ and a subgame perfect equilibrium strategy profile σ for $G^{\infty}(\delta)$ in which, on the equilibrium path (i.e. when no deviations have occurred), (D,L) is played in even periods and (U,R) is played in odd periods on the equilibrium path. (The initial period is period 0 so (D,L) should be played, etc.)

Note: in both parts, you are asked to *find* a discount factor and a strategy profile, not just appeal to an existence theorem.

5. Perfect Bayesian Signaling Equilibria

Consider a simple Spence signaling model in which education has only a signaling role. The productivity of a type θ worker is $\theta \in \Theta = [0,3]$. Types are continuously distributed with p.d.f. $f(\theta)$. The cost of education level q is $C(\theta, q) = \frac{q^2}{A(\theta)}$, where $A(\theta)$ is differentiable and strictly increasing. The outside payoff is $u_o(\theta) = 0$. Firms observe a worker's level of education but not productivity. The firms play a Bertrand wage game.

(a) Show that a separating PBE payoff for signaling types must satisfy a differential equation and hence show that there is a PBE in which the equilibrium payoff function is

$$U(\theta) = \frac{\int_0^\theta x A'(x)\, dx}{A(\theta)}.$$

(b) Is this (i) the unique separating PBE payoff? (ii) the unique PBE payoff? Explain.

Henceforth the outside payoff is $u_o(\theta) = \frac{1}{2}\theta$. That is, higher types have strictly better outside opportunities. Consider only the special case $A(\theta) = 3 + \theta^2$.

(c) Is the PBE in (a) still a PBE?

(d) Characterize the set of separating PBE.

(e) Which of these separating PBE satisfy the Intuitive Criterion?

6. Estate Auction

Two brothers have sentimental values of a painting by Mom. To keep things simple suppose that the market value is zero. Brother i, $i = 1, 2$ has a value $\theta_i \in [\alpha, \beta]$. Values are independently distributed with p.d.f $f(\theta)$ and c.d.f. $F(\theta)$.

An economist friend proposes that a mechanism be designed that (i) allocates the item to the brother who values it the most and (ii) leaves no surplus or deficit for the designer.

(a) To satisfy (i) explain why the expected total surplus is $\overline{S} = 2\int_{\alpha}^{\beta} \theta F(\theta) f(\theta) \, d\theta$.

(b) To satisfy (i) prove that the expected payoff to each brother depends only on the payoff to the lowest type $U(\alpha)$ and c.d.f. $F(\theta)$.

Henceforth assume that $\Theta_i = [0, 1]$ and $F(\theta) = \theta$.

(c) Solve for $U(0)$, the equilibrium payoff of the lowest type so that both (i) and (ii) are satisfied.

The economist friend proposes that each brother should submit a bid. The high bidder will win the painting and pay his winning bid to his brother.

(d) Obtain a differential equation for the equilibrium bid function $B(\theta)$. Check by substitution that there is a solution of the form $B(\theta) = \alpha\theta$.

(e) Solve for the equilibrium payoffs of the lowest and highest types.

(f) Does this auction satisfy (i) and (ii)?