# UCLA

## Department of Economics

## Ph. D. Preliminary Exam

## Micro-Economic Theory

(SPRING 2011)

**Instructions**:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

## 1. Consumer Problem with Quota

Consider a convenience store that sells two kind of (divisible) goods $x_1$ and $x_2$. $x_1$ and $x_2$ are on sale, but there is a limit on the amount each customer can purchase; the total amount of $x_1$ and $x_2$ cannot exceed some quota $Q > 0$. Let $p = (p_1, p_2) \gg 0$ be the price of $x_1$ and $x_2$ respectively. Alice is planning to spend $W > 0$ at this convenience store to purchase these goods. Let $u : \mathbb{R}^2_+ \to \mathbb{R}$ be Alice's utility function, which is differentiable. Answer the following questions.

(a) Write down Alice's utility maximization problem and, assuming interior solutions, write down the Kuhn-Tucker conditions for the problem. Explain why the Kuhn-Tucker conditions are necessary for any optimal solution.

(b) Suppose that $u(x_1, x_2) = x_1^{0.5} x_2^{0.5}$, $W = 80$, $Q = 40$, $(p_1, p_2) = (1, 5)$. Find Alice's optimal consumption $(x_1^*, x_2^*)$.

(c) Let $x(p, W, Q)$ be Alice's demand correspondence for this problem. Define Alice's indirect utility function by $v(p, W, Q) := u(x(p, W, Q))$. Show that $v$ is quasi-convex (i.e. the lower contour set is convex) in $(p, W)$ given $Q$ and quasi-convex in $Q$ given $(p, W)$. Is $v(p, W, Q)$ also quasi-convex in $(p, W, Q)$? Answer yes or no and explain why.

(d) Suppose that $u$ is strictly quasi-concave. Let $x^* \in \mathbb{R}^2_+$ be an optimal consumption vector for Alice given some $(p', W', Q') \gg 0$. Given $x^*$, consider the following cost minimization problem.

$$\min_{x \in \mathbb{R}^2_+} p' \cdot x \text{ s.t. } u(x) \geq u(x^*) \text{ and } x_1 + x_2 \leq Q'.$$

Show that $x^*$ is the unique solution for this problem.

**2. Excess Demand Function and Existence of Competitive Equilibrium.**

Consider a standard pure exchange economy $\mathcal{E}^{pure} = (\{X_i, \succeq_i, e_i\}_{i=1}^n)$ where $X_i = \mathbb{R}^L_+$, $\succeq_i$ is consumer $i$'s (rational and continuous) preference, and $e_i \in \mathbb{R}^L_+$ is consumer $i$'s initial endowment. Suppose that $\succeq_i$ is locally nonsatiated, strictly convex and monotone for every $i$ and $r = \sum_{i=1}^n e_i \gg 0$. Let $z : \mathbb{R}^L_{++} \to \mathbb{R}^L$ be the excess demand function defined by

$$z\,(p) := \sum_{i=1}^n [x_i(p, p \cdot e_i) - e_i]$$

where $x_i(p, p \cdot e_i)$ is consumer $i$'s (Walrasian) demand function. We know that there exists a competitive equilibrium when $z\,(p)$ satisfies the following five properties in $\mathbb{R}^L_{++}$.

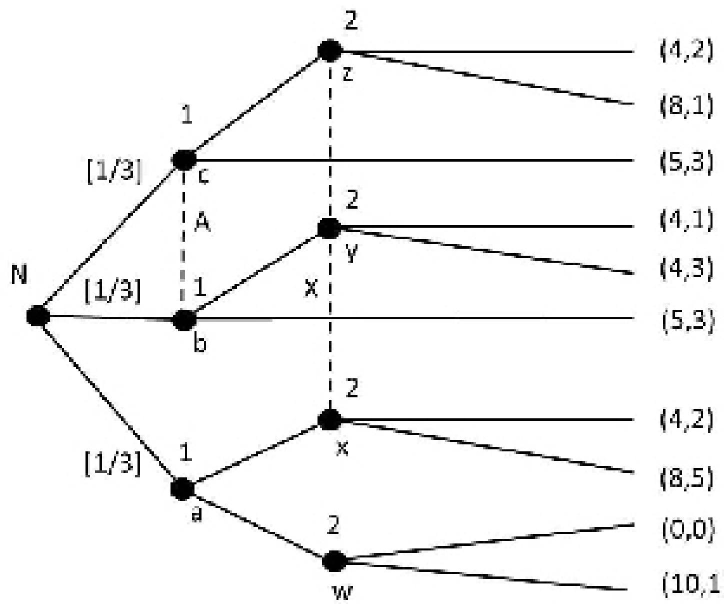
- **(I)** $z$ is continuous.
- **(II)** $z$ is homogeneous of degree 0.
- **(III)** $p \cdot z(p) = 0$ (Walras' Law)
- **(IV)** $z_\ell$ is bounded below for every $\ell$.
- **(V)** $\|z(p_n)\| \to \infty$ for any sequence of strictly positive prices in the price simplex $\{p_n\}_{n=1}^\infty \subset int\triangle$ such that $p_n$ converges to a boundary point of $\triangle$ ($\|\cdot\|$ is Euclidean norm).

Show that these five properties are indeed satisfied by $z(p)$ in this economy. State explicitly which assumptions are used in each step of your proof.

## 3. Equilibrium

For the game shown below, find:

1. all the sequential equilibria (actions and beliefs) in pure strategies
2. all the subgame perfect equilibria in pure strategies that are not sequential equilibria (if any)
3. all the Bayesian Nash equilibria in pure strategies that are not subgame perfect equilibria (if any)




In each case, explain thoroughly. (The given labeling of nodes and information sets may be useful.) In particular, if you find a subgame perfect equilibrium that is not a sequential equilibrium explain why not; if you find a Bayesian Nash equilibrium that is not a subgame perfect equilibrium, explain why not.

**4. Repeated Games** Consider the stage game $G$ below.

| | $C$ | $D$ |
|---|---|---|
| $C$ | $12, 12$ | $4, 8$ |
| $D$ | $0, 0$ | $6, 6$ |

1. Find the Nash equilibria in pure strategies and the minmax payoffs in pure strategies for $G$.

2. Make a careful sketch of the set of vectors $(x, y) \in \mathbb{R}^2$ for which there exists some discount factor $\beta < 1$ such that $(x, y)$ can be achieved as time-average payoffs of *some – not necessarily equilibrium – play* of the infinitely repeated game $G_\infty(\beta)$.

3. Let $W$ be the union of the point $(6, 6)$ with the line segment from $(8, 10)$ to $(12, 12)$. For what discount factors $\beta < 1$ is $W$ self-generating (in the time-average sense)?

**5. Auctions**

Buyers are risk neutral and values are independently and identically distributed with support $[0,1]$, c.d.f.$F(\theta)$ and p.d.f. $p(\theta)$. A single item is for sale using an auction in which the buyer with the high value is the winner. In the case of a tie the winner is selected randomly from the tying high bidders.

(a) Prove that the equilibrium expected payoff depends only on the c.d.f. and is independent of the payment rules.

(b) Hence prove revenue equivalence.

Two sisters have decided not to divide up their parents' estate. Instead they will allocate it using a sealed first price auction. The sister who makes the high bid and receives all the items in the estate pays her bid to the sister who makes the losing bid. (There is no third party collecting revenue.) The c.d.f. of the estate's value is uniform on $[0,1]$.

(c) Write down an expression for the equilibrium expected payoff . Differentiate this expression and hence show that the equilibrium bid function satisfies the following differential equation.

$$\theta\left[\theta B'(\theta)+2B(\theta)-\theta\right]=\theta^2 B'(\theta)+2\theta B(\theta)-\theta^2=0$$

(d) Solve for the equilibrium bid function.

## 6. A Chicken-Egg Problem

Each individual has a chicken with a 50-50 probability of laying either 0 or 2 eggs. (Probabilities are independent and eggs, once laid, are divisible.) Individual $i$'s quasilinear preferences are $v_i(z_i) + m_i = az_i - (b_i/2)z_i^2 + m_i$, and $z_i \geq 0$.

There are two sources of private information: knowledge of one's own tastes, here $b_i$, and knowledge of one's endowment — whether or not one's chicken has laid eggs. For (a) and (b), below, there is private information with respect to tastes but endowments are public information once they are realized. An insurance market (mechanism) maximizes the sum of individuals' expected utilities based on their reports.

(a) There are two individuals. Assuming $a = 2$, $b_1 = b_2 = 1$, find the efficient allocations of $(z_1, z_2)$ for the 4 equally likely expost outcomes, $(2, 2), (0, 0), (2, 0), (0, 2)$, (1's eggs, 2's eggs).

Suppose the mechanism calculates a money payment for the individual from whom it takes eggs equal to what that individual would receive in the expost price-taking equilibrium, with the opposite payment for the buyer. Each individual's net money payment is the probability weighted average of these expost transfers.

(b) If individual 1 were to report $b_1 = 1/2$ (when $b_1 = 1$) before endowments are realized, that would change the allocation and the net money transfers. Show that 1's expected utility including the transfers would *not* increase.

For the following, information about tastes is common knowledge: $a = 2$ and $b_i = 1$. There is, however, private information about individual endowments $(\omega_1, \omega_2)$, where $\omega_i \in [0, 2]$. If an individual's endowment is $\omega_i$ he can report $0 \leq r_i \leq \omega_i$ and keep $(\omega_i - r_i)$ to add to his final consumption

.

(c) When there are two individuals and $\omega_1 > \omega_2$, 1 will be the seller. Show that the scheme in (b) is not consistent with truthful reporting. Focus on individual 1 having $\omega_1 = 2$ when $\omega_2 = 0$ and individual 1 can report $r_1 = 1$ (after $\omega_2$ is realized).

(d) Is there a scheme that is efficient in the sense of maximizing the sum of utilities with respect to the $z$-commodity that would encourage truthful reporting no matter what the reported endowment of the other individual?