UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2009)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. First Welfare Theorem

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, \succsim_i, e_i\}_{i \in I})$ where $X_i = \mathbb{R}^L_+$, $\succsim_i$ is consumer i's (rational) preference and $e_i \in \mathbb{R}^L_+$ is consumer i's initial endowment.

(a) Define Walrasian (competitive) equilibrium in this economy.

(b) "As a consequence of Walras' Law, to verify that a price vector $p \in \mathbb{R}^L_{++}$ clears all markets, it suffices to check that it clears only $L-1$ markets". What assumption is behind this claim? Discuss briefly why it is needed.

(c) Prove that every Walrasian equilibrium allocation is Pareto efficient. If you need any additional assumption(s) to prove this claim, state them clearly.

2. Pure Exchange Economy with Cobb-Douglas Preference

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, \succsim_i, e_i\}_{i=1,2})$ with two consumers and two goods, where $X_i = \mathbb{R}^2_{++}$. Suppose that consumer i's preference is strictly convex, and can be represented by a Cobb-Douglas utility function $u_i(x_i) = x_{i,1}^{\alpha_i} x_{i,2}^{1-\alpha_i}, \alpha_i \in (0,1)$ for $i = 1, 2$. Answer the following questions.

(a) Write down consumer i's utility maximization problem given price vector $p \in \mathbb{R}^2_{++}$ and derive consumer i's demand $x_i(p, p \cdot e_i)$ for every $p \in \mathbb{R}^2_{++}$ (Do not use the formula!).

(b) Suppose that the profile of initial endowment vectors $e = (e_1, e_2) \in \mathbb{R}^2_{++} \times \mathbb{R}^2_{++}$ is a Pareto efficient allocation. Show that, if there exists a Walrasian equilibrium in this economy, it must be unique (after normalizing prices).

(c) Prove that there exists a competitive equilibrium in this economy for any $e \in \mathbb{R}^2_+ \times \mathbb{R}^2_+$ such that $e_1 + e_2 >> 0$. You may check if the assumptions for any existence theorem are satisfied for this economy (you don't need to prove the existence theorem itself), or you may prove existence directly.

3. Monopoly and product quality

Each consumer purchases either 0 or 1 unit of a commodity. A type t buyer's utility gain from paying a price r for a unit of quality q, $u_t(q,r)$, is a strictly increasing function of q and strictly decreasing function of r. Moreover the single crossing property holds, that is

$$-\frac{\partial u_t}{\partial q} / \frac{\partial u_t}{\partial r} > -\frac{\partial u_s}{\partial q} / \frac{\partial u_s}{\partial r} \text{ for any pair of types } s \text{ and } t > s.$$

The fraction of the population of type t is f_t, $t = 1,...,T$. The cost of producing a unit of quality q is cq. Any offer made to one customer must be made to all customers.

Let $r = R(q)$ be the indifference curve for type s through (q',r') and (q'',r'') where $q'' > q'$.

(a) With the help of a graph, explain why any type $t > s$ strictly prefers (q'',r'') and any type $t < s$ strictly prefers (q',r').

(b) Explain why the slope of $R(q)$ satisfies $R'(q) = -\frac{\partial u_s}{\partial q}(q,R(q)) / \frac{\partial u_s}{\partial r}(q,R(q))$.

Hence prove that the statement in part (a) is true.

(c) Explain briefly why, for the direct mechanism $\{(q_t,r_t)\}_{t=1}^{T}$ to be incentive compatible, $\{q_t\}_{t=1}^{T}$ must be increasing.

(d) Consider any direct mechanism $\{(q_t,r_t)\}_{t=1}^{T}$. Suppose that (i) $\{q_t\}_{t=1}^{T}$ is increasing and (ii) the "local downward constraints" are binding. Show that this mechanism is incentive compatible.

(e) Solve for the profit maximizing quality choices if $(f_1,f_2) = (\frac{3}{4},\frac{1}{4})$, $u_t(q,r) = \theta_t q - \frac{1}{2}q^2 - r$, where $(\theta_1,\theta_2) = (10,30)$ and $c = 2$.

4. First- and Second-Price Auctions

Consider an auction with two risk-neutral bidders bidding for a single object. The bidder's valuations (v_1, v_2) are independently, uniformly distributed on $[0, 1]$. The bids can take on any positive real number. Suppose first that the realizations of both valuations are known to the bidders and (after the auction) by the auctioneer as well. In the case of a tie, the object is allocated to the bidder with the higher valuation.

(a) Suppose that the object is allocated in a first-price auction. Characterize a Nash Equilibria.

(b) Suppose that the object is allocated in a second-price auction. Characterize an equilibrium in dominant strategies and one other equilibrium.

(c) Is either equilibrium in (b) trembling hand perfect?

Suppose next that the valuations are privately known to the bidders. It is common knowledge that the valuations (v_1, v_2) independently, uniformly distributed on $[0, 1]$.

(d) Characterize a Bayesian Nash Equilibrium of the Second Price Auction.

(e) Characterize a Bayesian Nash Equilibrium of the First Price Auction.

(f) Find an expression for the seller's expected revenue in the two auctions.

5. Second Price Principle

Two builders have valuations for each of two lots separately, but they also may have a valuation for adjacent lots that exceeds the sum of their separate valuations because with a bigger parcel they can build to sell to the rich who will pay more per square foot. Let α_i be i's valuation of the first lot, β_i the valuation of the second lot and σ_i the premium that i would pay above $(\alpha_i + \beta_i)$ to acquire both lots, $i = 1, 2$

(a) Let $(\alpha_1, \beta_1, \sigma_1) = (1, 2, 3)$ and $(\alpha_2, \beta_2, \sigma_2) = (2, 1, 1)$. Suppose the lots are auctioned separately according to the second price rule for each one. Show that if each bidder bids his true valuation for each lot, ignoring σ_i, the outcome is inefficient.

The efficiency/incentive properties of the second price auction assumes that valuations of objects are additive, i.e., $\sigma_i = 0$. Nevertheless, there is a principle underlying the efficiency and incentive compatibility of the second price auction which can be applied to the case of complementary valuations.

(b) Using this principle, formulate the analog for an efficient allocation of two complementary objects, i.e., when $\sigma_i \geq 0$. You will want to allow the builders to bid for each separately and also bid for the two together.

(c) Apply your auction rule to the example in (a) to illustrate that your auction encourages truthful bidding while also having the lots going to the highest valued user(s).

(d) Suppose the set of objects is $S = \{1, \ldots, K\}$. Bidder i's valuation of $T \subseteq S$ is $v_i(T)$, with $v_i(T) \geq v_i(T')$, $T' \subset T$, $i = 1, 2$. When objects are allocated efficiently (to be defined) based on reported valuations, what money payment scheme would encourage truthful reporting?

6. Repackaging Risk

The prototype is a model with two individuals. Each has an endowment W_i, an independent random variable taking values 0 and 1 with equal probability. A realization is $(w_1(s), w_2(s))$, $w_i(s) \in \{0, 1\}$, $s = 1, \ldots, 4$. A repackaging $X = (X_i)$ is feasible if $x_1(s) + x_2(s) = w_1(s) + w_2(s)$ for all s, $x_i(s) \geq 0$. Each has the same utility $U(X_i) = \alpha E(X_i) - \beta V(X_i)$, where $E(X_i)$ and $V(X_i)$ are the mean and variance of the random variable X_i and $\alpha, \beta > 0$. [For W_n, the sum of n independent W_i, $V(W_n) = n \cdot \frac{1}{2} \cdot \frac{1}{2}$]

There are two ways to enlarge the model: (I) increase the number of individuals to $2n$, n having an endowment whose realization is identical to individual 1 and n with realization identical to 2, above; (II) Increase the number of individuals to $2n$, each with identical independently distributed endowment.

(a) How do maximum per capita gains from trade vary in (I) and (II) as n increases?

Suppose two economies, A and B, differing in attitudes toward risk: for all individuals in A, $\alpha_A = 2$, $\beta_A = 1/2$ and for all in B, $\alpha_B = 1/2$, $\beta_B = 2$.

(b) Are there gains from trade between A and B when both are defined by (I) and $n = 1$? Endowment of both A and B governed by the same 4 states. If so, indicate why. As $n \to \infty$?

(c) Same as (b), except that A and B are described by (II).

(d) Suppose commodities are indivisible. Units can be split in half, but that is all. How would that effect your answers to (a) and (b)?