

**UCLA**  
**Department of Economics**  
**Ph. D. Preliminary Exam**  
**Micro-Economic Theory**  
(SPRING 2008)

**Instructions:**

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

### 1. Equilibrium with homothetic preferences

Consider a 2 period 2 commodity economy in which consumer  $h$ ,  $h=1,\dots,H$  has life-time utility function  $U^h = u(c_1^h) + \delta u(c_2^h)$  where  $c_t = (c_{t1}, c_{t2})$  is period  $t$  consumption and  $u(c_t) = \ln c_{t1} + \ln c_{t2}$ . The aggregate endowment is  $\omega = (\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22})$ . Commodity 2 is perishable. Commodity 1 ("corn") can also be planted and more corn produced in period 2. The technology is linear and for each unit of corn planted in period 1, the period 2 output is  $1 + \alpha$

- (a) Explain briefly why the assumption of homotheticity is helpful.
- (b) Characterize the Walrasian equilibrium if there is no production.
- (c) Under what assumptions about endowments and technology will there be no production in equilibrium? Henceforth assume that these conditions are violated so that there is production.
- (d) Suppose that the spot price and future spot price of commodity 1 are both 1. What will be the equilibrium interest rate?

### 2. Uncertain short takes

In each case you must present your reasoning.

- (a) A highly risk averse consumer will not invest in a risky asset unless the expected yield on a risky asset greatly exceeds the riskless yield. True or false?
- (b) A consumer with wealth  $W$  is offered the opportunity to win or lose  $D$  dollars (where  $D$  is small). A more risk averse agent will require the probability of winning to be shifted more in her favor before she will be willing to accept the gamble.
- (c) In an economy with  $S$  states and identical homothetic VNM utility functions and beliefs, risk is efficiently distributed even if the only assets are a riskless asset and a mutual fund consisting of the market portfolio. True or False?

### 3. An all-pay auction

Consider an all-pay auction among  $N$  bidders for two identical indivisible items. The rules of the auction are that the two high bidders each win one item and *all* bidders pay the amount of their own bid. Bidder's private valuations are drawn independently from the uniform distribution on  $[0, 1]$ .

Find a symmetric equilibrium in smooth strictly increasing bid functions.

### 4. Perfect Bayesian equilibrium

For the following game:

- (a) Consider a PBE in which Player 3 plays  $zA + (1-z)D$ . For what values of  $z$  will Player 1 always play  $D$ ? For what values of  $z$  will Player 2 always play  $D$ ? Use your answer to show that for all  $z$  the information set  $\{e, f\}$  will always be reached in equilibrium.
- (b) Find all the Perfect Bayesian Equilibria in which Player 3 does get to move (i.e., the information set  $\{e, f\}$  is reached in equilibrium).
- (c) Is there a Bayesian Nash equilibrium that is *not* perfect? Explain.

Note that the nodes are labeled for convenient reference.

