UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2008)

Instructions:

• You have 4 hours for the exam

• Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

• Use a SEPARATE bluebook to answer each question.
1. Equilibrium with homothetic preferences

Consider a 2 period 2 commodity economy in which consumer \( h, h=1,\ldots,H \) has life-time utility function \( U^h = u(c_1^h) + \delta u(c_2^h) \) where \( c_i = (c_{1i}, c_{2i}) \) is period \( i \) consumption and \( u(c_i) = \ln c_{1i} + \ln c_{2i} \). The aggregate endowment is \( \omega = (\omega_{11}, \omega_{12}, \omega_{21}, \omega_{22}) \). Commodity 2 is perishable. Commodity 1 (“corn”) can also be planted and more corn produced in period 2. The technology is linear and for each unit of corn planted in period 1, the period 2 output is \( 1 + \alpha \)

(a) Explain briefly why the assumption of homotheticity is helpful.

(b) Characterize the Walrasian equilibrium if there is no production.

(c) Under what assumptions about endowments and technology will there be no production in equilibrium? Henceforth assume that these conditions are violated so that there is production.

(d) Suppose that the spot price and future spot price of commodity 1 are both 1. What will be the equilibrium interest rate?

2. Uncertain short takes

In each case you must present your reasoning.

(a) A highly risk averse consumer will not invest in a risky asset unless the expected yield on a risky asset greatly exceeds the riskless yield. True or false?

(b) A consumer with wealth \( W \) is offered the opportunity to win or lose \( D \) dollars (where \( D \) is small). A more risk averse agent will require the probability of winning to be shifted more in her favor before she will be willing to accept the gamble.

(c) In an economy with \( S \) states and identical homothetic VNM utility functions and beliefs, risk is efficiently distributed even if the only assets are a riskless asset and a mutual fund consisting of the market portfolio. True or False?
3. An all-pay auction

Consider an all-pay auction among \( N \) bidders for two identical indivisible items. The rules of the auction are that the two high bidders each win one item and all bidders pay the amount of their own bid. Bidder's private valuations are drawn independently from the uniform distribution on \([0, 1]\).

Find a symmetric equilibrium in smooth strictly increasing bid functions.

4. Perfect Bayesian equilibrium

For the following game:

(a) Consider a PBE in which Player 3 plays \( zA + (1-z)D \). For what values of \( z \) will Player 1 always play \( D \)? For what values of \( z \) will Player 2 always play \( D \)? Use your answer to show that for all \( z \) the information set \( \{e, f\} \) will always be reached in equilibrium.

(b) Find all the Perfect Bayesian Equilibria in which Player 3 does get to move (i.e., the information set \( \{e, f\} \) is reached in equilibrium).

(c) Is there a Bayesian Nash equilibrium that is not perfect? Explain.

Note that the nodes are labeled for convenient reference.