UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(SPRING 2005)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Short-takes

(a) State and prove the First Law of Supply.

(b) Consider a constant returns to scale 2×2 economy in which both commodities A and B are produced. At the aggregate input endowment, commodity A is more input 1 intensive. Show that a change in preferences that results in an increase in the relative price of commodity A also leads to an increase in the relative price of input 1.

(c) In a two period economy there is no production but all goods can be costlessly stored. Use a simple model to discuss the conditions (if any) in which the equilibrium spot price of a commodity and the futures price of the same commodity will be the same.

2. Individual and aggregate risk

A consumer has a Von Neumann Morgenstern expected utility function $\nu(c_1, c_2) = c_1^{\alpha} c_2^{1-\alpha}$, where $\alpha \in (0,1)$. There are S states and the probability of state s is π_s . The state claims price vector is $p = (p_1, \ldots, p_S)$, where $p_s = (p_{1s}, p_{2s})$ and his wealth is $\overline{W}$.

(a) Let W_s be his wealth claims in state s so that $\sum_{s=1}^{S} W_s = \overline{W}$. Fix wealth claims and obtain an expression for his indirect utility function $U(W_1, \ldots, W_s)$.

(b) Under what conditions, if any, would he purchase only claims to state 1?

Henceforth consider an exchange economy where all consumers have the same beliefs and the same utility function given above. The aggregate endowment in state s is $\omega_s = (\theta_s \omega_1, \theta_s \omega_2)$, where $\sum_{s=1}^{S} \pi_s \theta_s = 1$.

(c) Compare the expected utility of consumers in this economy with the utility if there is no risk and the aggregate endowment is $\omega = (\beta_1, \beta_2)$.

(d) Suppose instead the aggregate endowment is $\omega_s = (\beta_1, \theta_s \beta_2)$. Again characterize equilibrium prices and compare expected utility with the outcome if there is no aggregate risk and the aggregate endowment is $\omega = (\beta_1, \beta_2)$.

3. Self-Confirming Equilibrium

Consider a three person centipede game in which player 1 can drop or pass, player 2 can drop or pass, and player 3 can drop or pass. If player 1 drops, the payoffs are (5,3,5); if player 2 drops the payoffs are (4,5,4), if player 3 drops the payoffs are (3,4,3) and if player 3 passes the payoffs are (8,6,8). What payoffs are possible in Nash equilibrium? In sequential equilibrium? Construct a self-confirming equilibrium that is NOT a public randomization over Nash equilibrium.

4. Brazil or the U.S.?

A long-lived government faces a short-run representative government employee. The government must choose whether to honor pensions (H) or not (N). At the beginning of the period, times are either good or bad. The probability that times are bad is 90%. In good times, pensions are always honored. In bad times they are honored or not depending on the government decision. There are two possible models of the employee: the informed employee who observes whether or not times are good or bad, and the uniformed employee who observed only whether pensions are honored or not. The choice of the employee is to guess whether or not her pension will be honored (H) or (N). The payoff of the employee is the sum of two parts: 1 if the pension is honored, 0 if it is not; and 1 for guessing right, 0 for guessing wrong. So guessing right when the pension is honored gives 2, and so forth.

Government payoffs in both good and bad times are:

	Guess H	Guess N
H	2	0
N	3	1

(a) Find the extensive and normal forms of the stage-game.

(b) For the long-run player, find the minmax, the static Nash, mixed precommitment and pure precommitment payoffs.

(c) Find the worst equilibrium for the long-run player, and describe in general terms the set of equilibrium payoffs for the long-run player. First assume that the employee is informed — that is can see whether or not times are good or bad.

(d) Find the best equilibrium for the government as a function of the discount factor δ.

Now assume that the employee cannot observe whether times are good or bad.

(e) Find the best equilibrium for the government as a function of the discount factor. Try to sustain the honoring of pensions.

5. A q-unit auction

A seller has q units for sale. There are $n > q$ buyers, each with a demand for at most one unit. The seller adopts an English auction: he asks each buyer for his bid b_i on how much the buyer is willing to pay, arrays the bids from highest to lowest, and then gives one unit to each of the q highest bidders at the price announced by the $q+1$ highest bidder. To illustrate, suppose $q = 4$, $n = 6$ and buyers' true valuations β_i are given by

$$(120, 100, 80, 50, 30, 10).$$

If the reported bids are $(100, 90, 70, 60, 30, 20)$, those buyers who bid $100, 90, 70, 60$ receive a unit and each pays 30.

(a) Show that the English auction gives each buyer his marginal product to the total gains from trade. Use the specific example to verify.

(b) Show that the English auction makes the truthful bidding function $b_i(\beta_i) = \beta_i$ a dominant strategy for each buyer.

(c) Assume the seller's cost for all units he can supply is 0. What is the seller's marginal product? Does the seller receive his marginal product in the English auction?

(d) In an attempt to gain more, the seller names a reserve price $\bar{p}$ and modifies the auction so that the winning bids are those that are greater than or equal to the max of $\bar{p}$ and the $q+1$ highest price. Show that this does not effect the incentive to bid truthfully.

(e) If the seller knows that the buyers' valuations are as in the example above, what value should the seller set for $\bar{p}$? (Draw a picture.) Is the outcome of the auction efficient? [Note: The seller is unable to practice price discrimination, e.g., because although the seller knows the distribution, the seller does not know who has which valuation.]

(f) Modify the example above of buyers' valuations so that the seller would not do better by setting a reserve price. Compare it to the outcome in (a).

6. Bus Service

A town is deciding on the number and quality of buses to purchase. The total money cost of bus service is $c(f, \ell)$ where f is the frequency of service (a proxy for the number of buses purchased) and ℓ is the luxuriousness of the ride (which increases the cost per bus). Each individual's utility is $v_i(f, \ell) + m_i$, $i = 1, \ldots, n$.

(a) What is the criterion for a Pareto optimal decision? What are the first-order conditions for a Pareto-optimal decision?

(b) Under what conditions on v_i and c are the first order conditions in (a) sufficient for optimality?

(c) Write the conditions for the Lindahl equilibrium method of making the decision.

(d) Show that Lindahl equilibrium leads to a Pareto optimal decision *without* assuming the conditions in (b).

Recognizing the strategic shortcomings of (d), the town members adopt the following decision rule: Anyone can propose a (f, ℓ, t) where t is the cost share to each individual of the decision (f, ℓ). Of course, the per person share must be such that $nt = c(f, \ell)$. The proposal with the most votes wins. There are two types of individuals. The poor who value frequency over luxury have $v_P(f, \ell) = 3(f^{2/3} + \ell^{1/3})$ and the rich who do not take the bus very often but want to ride in comfort when they do. Their $v_R(f, \ell) = 3(f^{1/3} + \ell^{2/3})$. There are n_P poor and n_R rich and $n_P > n_R$. The cost function is $c(f, \ell) = f^2 + \ell^2$.

(e) Find the winning (f, ℓ, t)? Is it Pareto optimal?