July 1, 2004

UCLA Department of Economics

First-Year Core Comprehensive Examination in MICROECONOMICS

Spring 2004

Instructions: *DO 5 PROBLEMS.*
(If you do more than 5 problems, only the first 5 will be graded.)
Answer each question in a separate answer book AND write the question number on the outside of each book.

You have 4 hours

1. Homothetic Preferences

a) Define a homothetic utility function and sketch a proof that, for a homothetic consumer, the income elasticity of demand is unity.

b) In a 2 commodity endowment economy, consumer h, $h = 1, 2$, has a homothetic utility function U^h. The aggregate endowment is (ω_1, ω_2). What can you say about the Walrasian equilibrium price vector

   i) if $U^1 = U^2$?

   ii) if $U^1 \neq U^2$?

c) Suppose that only consumer 1 has a strictly positive endowment of commodity 1 and that both have Cobb-Douglas utility functions. Consumer 1 sets a relative price of p_1. What utility level can consumer 1 achieve achieve assuming that consumer 2 is a price taker? What price should consumer 1 set?

d) If consumer 2s utility function is not Cobb-Douglas is it necessarily the case that consumer 1 is strictly better off if he is a price setter than he would be in Walrasian equilibrium?

2. Robinson Crusoe economy with uncertainty

There are 2 periods, 2 states and 1 commodity. The probability of state s is π_s. In period 1 Robinson Crusoe's endowment is ω_1 . In period 2 it is ω_2 (in each state). There are also types of firm. A type A firm can produce αx units of output in period 2 (in each state) using, as input, x units of the commodity in period 1. A type B firm can produce βy units of output in period 2 if state 1 occurs using y units of the commodity in period 1. If state 2 occurs, the output of a type B firm is zero.

Robinson Crusoe has a Cobb-Douglas utility function

$$U(c_1, c_{21}, c_{22}) = \ln c_1 + \pi_1 \ln c_{21} + \pi_2 \ln c_{22}$$

a) Suppose initially that output of type B firms is zero. Solve for the optimal production plan x^* of type A firms. Under what condition on α will x^* be strictly positive?

b) If this condition holds, what condition must hold for it to be optimal for type B firms to produce nothing?

c) What role does risk aversion play in these conclusions?

d) Suppose that optimal production plans x^* and y^* are both strictly positive. Setting the first period price equal to 1, what will be the Walrasian equilibrium state contingent prices for goods delivered in period 2?

e) What will be the market value of the firms (in terms of probabilities and inputs) (i) prior to the purchase of inputs? (ii) after the inputs have been purchased?

3. An Auction

3 bidders participate in a sealed-bid auction for two identical objects. Bidder n's private value is s_n; these values are drawn independently from the uniform distribution on $[0, 1]$. The highest and second-highest bidders each win one of the objects, and each pays his/her own bid. [Bids are required to be non-negative.]

Derive carefully the unique symmetric Bayesian Nash equilibrium in smooth, strictly increasing strategies (bidding functions).

4. Trial or Settlement?

John fell on the sidewalk outside Mary's Diner. Everyone agrees that the value of the injuries John sustained is \$100,000, but they do not agree whether Mary is liable for these injuries. John has therefore filed suit, and the trial is scheduled to begin tomorrow. If Mary is held liable, she will have to pay John \$100,000; if Mary is held not liable she will not have to pay John anything. The outcome of trial will depend on whether the jury believes Mary took proper care of her sidewalk. If Mary has a good record of care, she is strong and will win the case 80% of the time; if Mary has a bad record of care, she is weak and will win the case 20% of the time. If the case goes to trial, *both* John and Mary will have to pay costs and fees of \$10,000 whatever the outcome. (So if Mary wins the case, her payoff is -\$10,000 and John's payoff is -\$10,000; if Mary loses the case, her payoff is -\$110,000 and John's payoff is \$90,000.)

However, Mary still has the opportunity to make a settlement offer, which can be anything between \$0 and \$100,000. If John accepts Mary's offer, it will be implemented; if John rejects Mary's offer the case will go to trial.

Mary knows whether she has a good record. John does not know whether Mary has a good record or a bad record, but he thinks the probabilities of each are .5 (and this fact is common knowledge).

a) Model this situation as a game in extensive form with incomplete information. Sketch the game tree. [Assume Mary always makes an offer, since offering \$0 has the same effect as making no offer.]

b) Find all the pooling Perfect Bayesian Equilibria in pure strategies.

c) Find all the separating Perfect Bayesian Equilibria in pure strategies.

In b), c) you may assume John accepts whenever he is indifferent. Be sure you specify strategies and beliefs, as well as outcomes.

5. Competition with Fixed Costs

Fixed costs leading to decreasing average costs are usually regarded as inimical to competition, but consider the following. There are two commodities, bottled water and money; and there are two identical producers and two identical consumers. Producer $j = 1, 2$ supplies *delivered* water to consumer $i = 1, 2$ according to a cost function (in terms of money) given by

$$c(q_{ji}) = \begin{cases} 0 & \text{if } q_{ij} = 0 \\ q_{ji} + 4 & \text{if } q_{ij} > 0 \end{cases}$$

where $q_{ji} > 0$ is the quantity that j delivers to i, i.e., there is a unit constant marginal cost of bottling the water and 4 is the fixed delivery charge that is independent of the quantity delivered. Consumer $i = 1, 2$ has (quasilinear) utility for water $v(q_i) = 10q_i$, $0 \leq q_i \leq 1$.

a) Verify that the cost function for delivered water is not convex. Given that the price of money is unity and the price/unit of water is p, demonstrate that a price-taking equilibrium does NOT exist.

b) Calculate the marginal product of each participant and show that they add up to the maximum total gains from trade. Would these marginal products change if there were n consumers and n firms?

c) Specify a *two-part* pricing schedule for delivered water such that if the two producers and two consumers take that schedule as given, there is an equilibrium in which each receives its marginal product. Would two-part pricing be an equilibrium with large numbers of producers and consumers?

d) Modify the cost function above so that there is a fixed cost for *producing* water, independently of who gets it, but no costs to delivering it. Suppose

$$c(q_j) = \begin{cases} 0 & \text{if } q_j = 0 \\ q_j + 4 & \text{if } q_j > 0 \end{cases}$$

Demonstrate that, as in a), there is no price-taking equilibrium price p/unit when there are two producers and two consumers. What if there were *many* consumers?

6. Franchise Contracting

A franchisor of restaurants contracts with each of two franchisees. The productivity of each franchise in terms of gross *money value* of output is $y_i(e_0, e_i)$, where e_0 is the effort by the franchisor and e_i the effort by the franchisee, $i = 1, 2$. Notice that the franchisor's effort is a kind of public good since it is simultaneously available to both franchisees. The net return to each party is: $u_j(m_j, e_j) = m_j - c_j(e_j)$, $j = 0, 1, 2$, where c_j is j's cost of effort function and m_j is the money payment received by j. Assume concavity for y_i and convexity for c_j. Total money payments are equal to the total money value of output:

$$(*) \qquad m_1 + m_2 + m_3 = y_1(e_0, e_1) + y_2(e_0, e_2)$$

Contracts cannot be written on effort, e.g., because effort is not verifiable.

a) Suppose the franchisor contracts with each franchisee i to pay $y_i/2$. How would the resulting equilibrium levels of effort compare with the optimal levels of effort? (Besides a description of the (calculus) conditions for equilibrium, your answer should include the conditions for optimal effort by the three parties with respect to the two franchises.)

b) Franchisee i is a "residual claimant" if i gets all the output attributable to its effort, i.e.,

$$(1) \qquad m_i = y_i(e_0, e_i) - y_i(e_0, 0).$$

Similarly, the franchisor is a residual claimant when

$$(2) \qquad m_0 = y_1(e_0, e_1) + y_2(e_0, e_2) - [y_1(0, e_1) + y_2(0, e_2)].$$

Ignoring the feasibility condition $(*)$, show that if all parties are residual claimants, all parties will be led to supply optimal levels of effort.

c) The franchisor can modify the residual claimant scheme for the franchisees so that m_i equals the amount in (1) *plus* a term, call it L_i, which does not depend on e_i. Show that L_1 and L_2 can be chosen such that the feasibility condition $(*)$ is satisfied at the equilibrium. [Note: The franchisor can compute L_i based on hypothesized values for e_j, $j \neq i$ and e_0.]