MICROECONOMICS QUALIFYING EXAM - FALL 1999

Time allowed four (4) hours. You must attempt five 5 questions.

1. State claims markets with trade in two periods.

There is one commodity. Individual h, $h=1,\ldots,n$ has a period 1 endowment of ω_1^h and an uncertain period 2 endowment of $(\omega_{21}^h, \omega_{22}^h)$. The aggregate endowment is ω in each state and each date. The period 1 commodity can be costlessly stored for consumption in period 2. Individual h has *VNM* utility function

$$U(c^h) = v(c_1^h) + \delta(\pi_1 v(c_{21}^h) + \pi_2 v(c_{22}^h)),\ \delta < 1$$

(a) Characterize as completely as you can the Pareto efficient allocations and the state Walrasian equilibrium price vectors in this economy.

(b) The futures price is the price which provides a unit tomorrow independent of the state. What is the equilibrium futures price? What is the equilibrium interest rate?

(c) Suppose that the aggregate endowment of the commodity in period 1 is made smaller. Everything else remains as before. What effect would this have on the Pareto efficient allocations? What effect would this have on the period 2 state claims price ratio p_{21} / p_{22}?

(d) Same question but this time the aggregate endowment is made high enough in period 1 that at least some people will store.

(e) Return to the assumptions in parts (c) and (d) and say what you can about the effect on the interest rate in each case.

(f) How would your answers to parts (a) and (b) change if each individual had the same beliefs but different *VNM* preferences?

2. Choosing capacity in the long-run

(a) There are n plants in a firm. Plant i can produce x_i units of machinery at a cost of $C_i(x_i) = \dfrac{x_i^2}{2\alpha_i}$. Show that if the firm wishes to produce x units in total, the minimized total cost is $C(x) = \dfrac{x^2}{2\Sigma}$, where $x = \sum_{i=1}^{n} x_i$ and $\Sigma = \sum_{i=1}^{n} \alpha_i$.

The firm uses its stock of machinery to produce output q. Total output capacity in period t, q_t can be augmented next period by $\Delta q = \theta x_t$, where x_t is the number of new machines, produced at a cost of $C(x_t)$. Capacity depreciates at a rate γ so that

$$q_{t+1} = q_t + \theta x_t - \gamma q_t .$$

The firm's revenue in period t is $R(q_t)$.

(b) Suppose that the firm decides to increase capacity by dq and maintain it there forever. Explain why present value of the marginal revenue stream

$$PV\{\text{Marginal Revenues}\} = R'(q_1)dq(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots) = R'(q_1)\frac{dq}{r}$$

Also,

$$PV\{\text{Marginal Costs}\} = C'(x_1)dx + \gamma C'(x_1)dx(\frac{1}{1+r} + \frac{1}{(1+r)^2} + \ldots) = C'(x_1)(1+\frac{\gamma}{r})dx .$$

(c) Hence or otherwise obtain necessary conditions for q and x to be at their long run stationary state levels.

(d) Suppose demand is given by $p_t = a - bq_t$ and the cost function for new capacity as given by part (a). Solve for the long run stationary state capacity.

(e) Suppose instead that each of the n plants is a separate firm. Use a similar approach to determine the Walrasian long run equilibrium steady state.

(f) What difference would it make if the machinery producing firms were to sell new machines to downstream producers of the final product?

3. Mechanism Design

The U.S. government is considering converting a certain military technology for civilian use; doing so will require conversion costs of C. The government knows that one or both of Firms A and B will be able to use this technology; the probabilities are:

1. $p(A \text{ only}) = .4$
2. $p(B \text{ only}) = .3$
3. $p(A \text{ and } B) = .3$

Each firm knows whether it will be able to use the technology (but neither knows whether the other firm will be able to use it).

If the military technology is converted to civilian use, and only one of the firms is able to use the technology, that firm will make a profit of \$100 million. However, if both firms are able to use the technology, Firm A's superior marketing abilities mean its profit will be \$40 million while Firm B's profit will be only \$20 million.

All this is common knowledge.

The government asks you, as an expert in game theory and procurement, to design a mechanism for deciding whether to convert the military technology to civilian use and how to apportion the cost.

(a) For what values of the cost parameter C is there a socially efficient, individually rational, incentive compatible mechanism for making this decision?

(b) For these values of C, describe such a mechanism and verify that it has the required properties.

4. Repeated Games

Below is a 2-person game with 3 strategies for each player.

(a) Find 2 (pure strategy) Nash equilibria for this game.

Now consider the 3-fold repetition of this game, with discount factor δ, taken as a parameter.

(b) Find a $\bar{\delta}$ with $0 < \bar{\delta} < 1$ such that

(i) For $\delta > \bar{\delta}$, there **is** a subgame perfect equilibrium (in pure strategies) in which (U, L) is played in the first round.

(ii) For $\delta < \bar{\delta}$, there is **no** subgame perfect equilibrium (in pure strategies) in which (U, L) is played in the first round.

(c) Would your answer to the preceding part be different if *mixed strategies* were allowed?

	L	C	R
U	$(4,4)$	$(0,5)$	$(0,0)$
M	$(5,0)$	$(1,3)$	$(0,0)$
D	$(0,0)$	$(0,0)$	$(3,0)$

5. Allocating risk

Consider a pure exchange economy with a continuum of consumers, two dates $t = 0, 1$, two states of nature $\omega = \omega_1, \omega_2$ (revealed to consumers between dates 0 and 1), and a single commodity available at each date-event. All consumers have the same utility function given by

$$U(x_i) = \ln x(0) + \pi \ln x(t, \omega_1) + (1 - \pi) \ln x(1, \omega_2)$$

where π is the probability that state ω_1 occurs. Consumers of type a have endowment

$$w_a = (w_a(0), w_a(1, \omega_1), w_a(1, \omega_2)) = (2, 2, 2)$$

and consumers of type b endowment

$$w_b = (w_b(0), w_b(1, \omega_1), w_b(1, \omega_2)) = (2, 10, 1)$$

A fraction $\lambda \in (0, 1)$ of consumers are of type a and $1 - \lambda$ are of type b. Normalize prices such that $p(0) = 1$.

(a) Show that the Arrow-Debreu prices for this economy are given by

$$p(1, \omega_1) = \frac{\pi}{5 - 4\lambda} \qquad \text{and} \qquad p(1, \omega_2) = \frac{2(1 - \pi)}{1 + \lambda}$$

(b) Assuming that $\lambda = 1/2$ and $\pi = 1/2$, solve for the Arrow-Debreu equilibrium prices and equilibrium allocation.

(c) Continuing to assume that $\pi = 1/2$, suppose that consumers at date 0 can choose whether their endowment will be w_a or w_b. (Interpretation: each consumer chooses whether to be in the riskless occupation a or the risky occupation b.) Solve for the equilibrium fraction λ of consumers who choose endowment w_a, and find the corresponding Arrow-Debreu equilibrium prices and allocation.

(d) Compute equilibrium net trades for the allocations of parts (b) and (c), and compare.

6. Labor markets

Consider a production economy with a continuum of consumers, indexed by the interval $I = [0, 1]$ with Lebesgue measure. Consumer i has endowment $w_i = (0, 0, 0, 0)$ and utility function

$$U_i(x_i) = i[x_{i1}x_{i2}(24 + x_{i3})] + (1 - i)[x_{i1}x_{i2}(24 + x_{i4})]$$

Commodity 1 is produced using the activity vector $y_1 = \lambda(1, 0, -2, 0)$ where $\lambda \geq 0$. Commodity 2 is produced using the activity vector $y_2 = \lambda(0, 1, 0, -1)$ where $\lambda \geq 0$. Commodities 3 and 4 represent labor supplied by the consumers. x_{i3} and x_{i4} must be non-positive and satisfy the restrictions

$$-24 \leq x_{i3} + x_{i4} \leq 0 \quad \text{and} \quad x_{i3}x_{i4} = 0$$

(i.e., consumer i can work no more than 24 hours and can work in at most one "industry"). Normalize prices so that $p_3 = 1$.

Solve for the Walrasian equilibrium allocation and the Walrasian equilibrium prices. Which consumers choose to work in industry 1? In industry 2? Verify explicitly that demand equals supply in all four markets.