UCLA Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

FALL 1998

Instructions:

- You have **4** hours for the exam. The first 10 minutes are for reading only.
- Answer any **5** of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions. If you do, only the first five will be graded.
- Use a SEPARATE bluebook to answer each question.

Part I

1. Constant Returns to Scale Economy

Output of the two products in an economy is produced from labor and capital according to the production functions

$$x_1 = \sqrt{L_1 K_1} \qquad \text{and} \qquad x_2 = \left(\sqrt{L_2} + a\sqrt{K_2}\right)^2, \qquad a > 0.$$

The aggregate supply of labor and capital $(\bar{L}, \bar{K}) = (100, 100)$.

(a) Show that the set of feasible outputs X is convex.

(b) Let x^0 and x^1 be two points on the production possibility frontier. For what values of the parameter a is the convex combination $(1-\lambda)x^0 + \lambda x^1$, $0 < \lambda < 1$, (i) on the frontier or (ii) in the interior of X?

(c) Suppose $a < 1$. At world prices $p = (p_1, p_2)$, both commodities are produced. If the price of commodity 1 rises, explain carefully how (i) equilibrium outputs and (ii) the equilibrium wage rental ratio will be affected.

(d) Would your answer change if $a \geq 1$?

(e) Suppose $a = 2$. Might it be efficient for the economy to specialize in the production of commodity 1? (A brief discussion only is expected.)

2. Risk

The risky prospect $\tilde{x} = (x_1, x_2; \pi_1, \pi_2)$ yields the outcome x_s with probability π_s, where $\pi_1 + \pi_2 = 1$. A consumer is faced with choosing between $\tilde{x}$ and its expectation $\bar{x} = \pi_1 x_1 + \pi_2 x_2$ with certainty.

(a) If $\tilde{x}$ is a risky bundle of consumption goods, show that a consumer with utility function $U(x)$ will prefer $\bar{x}$ if $U(\cdot)$ is strictly concave and $\tilde{x}$ if $U(\cdot)$ is strictly convex.

(b) There are two commodities c_1 and c_2. If consumer h has income I and faces a price vector p, and his utility function is $U_h(c) = c_1^{\alpha} c_2^{\beta}$, $\alpha, \beta > 0$, $\alpha + \beta \leq 1$, solve for his indirect utility function.

(c) Under what conditions, if any, will this consumer strictly prefer the risky income $\tilde{I}$ to its expectation $\bar{I}$.

(d) Suppose income and the price of commodity 2 are given but the price of commodity 1 is uncertain. Under what conditions, if any, will this consumer striclty pefer the risky price $\tilde{p}_1$ to its expectation $\bar{p}_1$?

(e) Let $\Pi_f(p_1)$ be the maximized profit of firm f. Prove that this function is convex.

(f) "In this economy, there are conditions under which both consumers and firms strictly prefer price uncertainty. Therefore society gains if the monetary authority creates price instability." True or False? Explain.

Part II

3. Mechanism Design

The government is considering whether or not to develop a new technology for producing airplane fuselages. The technology is potentially useful to the only 2 remaining US airframe manufacturers, Lockheed and McDonnell-Douglas. The potential value of this technology is \$50 Million to Lockheed and \$70 Million to McDonnell-Douglas. However, both Lockheed and McDonnell-Douglas have already researched alternative technology; if either company has been successful, that company will have no use at all for the government technology. [Once developed, the government technology will become public and can be used by either or both firms without cost. However, private technology developed by either firm will remain private and not usable by the other firm.] Each firm knows whether it has been successful, but no one else does; *ex ante*, it is common knowledge that the probability of success by a given firm is 1/2. The cost of developing the government technology is C; the government will develop the technology exactly if the two firms are willing to share the cost. For what values of C does there exist a socially efficient, individually rational, incentive compatible mechanism for deciding whether or not the government should develop the technology and how much each firm should be charged? [Each firm is free not to participate; the government is allowed to treat the two firms differently.]

4. Repeated Games

ROW and COLUMN play the following stage game 3 times.

	L	C	R
U	2 2	7 12	1 1
M	12 7	0 0	1 1
D	1 1	1 1	0 20

(a) What is the highest total payoff (not discounted) that COLUMN can achieve in any Nash equilibrium in pure strategies? Find a Nash equilibrium that achieves this payoff.

(b) What is the highest total payoff (not discounted) that COLUMN can achieve in any subgame perfect equilibrium in pure strategies? Find a subgame perfect equilibrium that achieves this payoff.

Part III

5. Labor Supply

Consider a perfectly competitive economy with a continuum of consumers, indexed by $I = [0, 1]$ with Lebesgue measure. There are three commodities: 1 (labor); 2 (another input); and 3 (a produced commodity). Assume that for consumer i, labor supply $x_{i1} \in [-24, 0]$ and that consumption of commodities 2 and 3 must be non-negative.

Assume that consumer $i \in I$ has utility function

$$u_i(x_i) = (24 + x_{i1})x_{i2}x_{i3}$$

and endowment

$$w_i = (0, 8i, 0)$$

Production, which is subject to constant returns to scale, can be any non-negative scalar multiple of the activity vector

$$y = (-2, 0, 1)$$

Let $p = (p_1, p_2, p_3)$ denote the vector of commodity prices.

(a) Give the first-order conditions for utility maximization for consumer i and derive the consumer's supply of labor and demand for commodities two and three as a function of prices and the index i.

(b) Find the Walrasian equilibrium allocation and prices for this economy. Verify that this allocation is feasible (including satisfaction of the consumption set restrictions for each consumer). Show how the aggregate allocation of each commodity can be interpreted as the per capita allocation, averaged over all consumers.

(c) Is the Walrasian allocation Pareto optimal? in the core of the economy? Give a brief justification for your answer.

(d) Is the Walrasian allocation the only allocation in the core of this economy? Interpret this fact.

6. Insuring for Long-Term Care

Consider an economy with 3 consumers and 3 dates, indexed by $t \in \{0, 1, 2\}$. There is one physical commodity (corn) distinguished by date and event. For consumer i, let $x_i(t, a_t)$ denote consumption and $w_i(t, a_t)$ endowment at date-event (t, a_t).

Unless otherwise specified, each consumer is endowed with two units of corn at every date and event. At date 1, one consumer is chosen at random (with equal probability for all three consumers), and his endowment is reduced to zero for dates 1 and 2. At date 2, a different consumer is chosen at random (with equal probability for both), and his endowment reduced to zero for date 2. Assume consumer i has utility function

$$U_i(\cdot) = \sum_{t=0}^{2} \sum_{a_t \in f_t} \pi(a_t) \ln(x_i(t, a_t))$$

where f_t denotes the partition of events observed at date t and $\pi(a_t)$ is the probability event $a_t \in f_t$ occurs. (There is no time discounting.)

(a) Suppose Arrow-Debreu contingent commodities for every date-event are available for trade at date 0. Setting the price of corn at date 0 equal to 1, compute the Arrow-Debreu equilibrium allocation, net trades and prices for this economy.

(b) Suppose contingent commodities are available only one period in advance. Does a Radner equilibrium exist for this economy which can attain the allocation of part (a)? If so, find the prices and describe the trades consumer 1 makes over time. If not, explain why an equilibrium fails to exist.

(c) Suppose at date 0 an insurance company offers a long-term contract which will pay the consumer an amount equal to the per capita endowment of the economy whenever the consumer's endowment equals 0. Payments by the consumer to the insurance company for this contract are to start at date 1 but will be waived whenever the consumer has 0 endowment. Find the appropriate level of payments to assure that, with every consumer purchasing this contract, the equilibrium allocation of part (a) is attained and the insurance company earns zero profits.