UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2014)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. First Welfare Theorem

Let $\mathcal{E}^{pure} = (\{X_i, \succeq_i, e_i\}_{i \in I})$ be the standard pure exchange economy with free disposal, where $X_i = \mathbb{R}^L_+$ and $\succeq_i$ is locally nonsatiated for every $i \in I$. Answer the following questions.

(a) Define Walrasian equilibrium and Pareto efficient allocation in this economy.

(b) Prove that every Walrasian equilibrium allocation is Pareto efficient.

(c) Suppose that $I = \{1, 2, 3\}$. Suppose that consumer 1 and consumer 2 decide to trade exclusively with each other, effectively excluding consumer 3 from any exchange. Consumer 1 and 2 negotiate to come up with a pair of consumption vectors $x'_1, x'_2 \in \mathbb{R}^L_+$ such that $x'_1 + x'_2 \leq e_1 + e_2$. Of course consumer 3 just consumes her endowment e_3 (or a part of it). Let $(x^*, p^*) \in \mathbb{R}^{3L}_+ \times \mathbb{R}^L_+$ be any Walrasian equilibrium that would have realized if every consumer can participate in the market. Clearly consumer 3 is always (weakly) worse off by consuming e_3 rather than x^*_3. But is it possible that consumer 1 and 2 are better off negotiating with each other, i.e. $x'_i \succeq_i x^*_i$ for $i = 1, 2$ and $x'_i \succ_i x^*_i$ for $i = 1$ or 2? If so, find such an example. If not, explain why.

2. Pareto Efficiency and Externality

We consider a pure exchange economy $\mathcal{E}^{ext}$ with consumption externality, where consumer 1's utility is directly affected by other consumers' consumptions. Consumer 1's preference can be represented by a continuous utility function $u_1(x) = f(x_1) - \sum_{i \neq 1} g_i(x_i)$, where f is increasing $(x_1'' \gg x_1' \Rightarrow f(x_1'') > f(x_1'))$ and concave and g_i is increasing and convex. The preference of consumer $i\,(\neq 1)$ is represented by a usual utility function $u_i(x_i)$, which is increasing and concave.

(a) Let $U = \{u \in \mathbb{R}_+^I : \exists x \text{ feasible}, u \leq u(x)\}$ be the utility possibility set. Show that U is closed and convex.

(b) Show that a feasible allocation x in this economy $\mathcal{E}^{ext}$ is Pareto efficient if and only if it maximizes the weighted sum of utilities with respect to some weight vector $(a_1, ..., a_I) \in \mathbb{R}_{++}^I$.

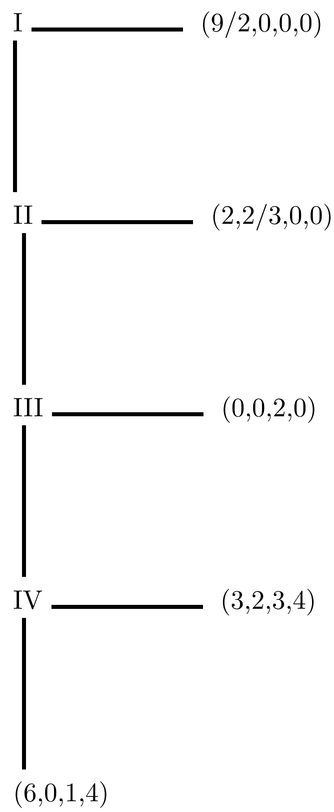
(c)Consider the following two-good pure exchange economy with consumption externality: $I = \{1, 2\}$, $u_1(x) = \ln x_{1,1} + \ln x_{1,2} - x_{2,1}$, $u_2(x_2) = \ln x_{2,1} + \ln x_{2,2}$, and $e_1 = e_2 = (1/2, 1/2)$. The definition of Walrasian equilibrium $(x^*, p^*) \in \mathbb{R}_+^4 \times \mathbb{R}_+^2$ is the same as usual, except that x_1^* solves $\max_{x_1 \in \mathbb{R}_+^2} u_1(x_1, x_2^*)$, s.t. $p^* \cdot x_1 \leqq p^* \cdot e_1$ given x_2^*. Characterize the set of Pareto efficient allocations in $\mathbb{R}_{++}^4$ and show that every Walrasian equilibrium allocation in $\mathbb{R}_{++}^4$ is Pareto inefficient.

3. Subgame Perfect Equilibrium

The diagram below shows a 4-player game.

(a) Find *all* the subgame perfect equilibria in *pure strategies*.

(b) Find *all* the subgame perfect equilibria (if any) in which player I plays a completely mixed strategy.



4 Public Good Provision

The government must decide whether to build a project that is of potential value to two firms. The cost of the project is c; the value to firm 1 is either 1 or 0, the value to firm 2 is either 2 or 0; in each case the probability of a positive value is p (where $0 < p < 1$) and the probabilities are independent. Whenever the government decides to build the project it will divide the cost c between the firms but will *never make a profit or provide a subsidy.*

The government wants to use a socially efficient mechanism: that is, a mechanism that causes the project to be built if and only if the cost is less than the total value to the firms. (To avoid complications we will ignore cases where the cost might be exactly equal to the total value to the firms.)

Notice that this is *not* a symmetric problem, so the mechanism(s) need not be symmetric either.

(a) If $2 < c < 3$, find a socially efficient mechanism that is incentive compatible and (interim) individually rational for the firms. (That is, the firms are willing to participate in the mechanism after they know their true values.)

(b) If $1 < c < 2$, find a socially efficient mechanism that is incentive compatible and (interim) individually rational for the firms.

(c) If $0 < c < 1$, find the region in cost c and probability p space for which there is a a socially efficient mechanism that is incentive compatible and (interim) individually rational for the firms. In that region find such a mechanism.

5. Signaling with Outside Opportunities

Consider the following simple Spencian signaling model. The set of types is $\Theta = \{\theta_1, \theta_2, \theta_3\} = \{1, 3, 4\}$, which are equally likely. A type t worker has a marginal product of θ_t. The cost of signaling at level q for type θ_t worker is $C(\theta_t, q) = A(\theta_t)\, q$, where $A(\theta_1) = \frac{1}{2}$, $A(\theta_2) = \frac{1}{3}$, $A(\theta_3) = \frac{1}{10}$.

A type θ_t worker has an outside payoff (self-employment) of $u_o(\theta_t)$, where $u_o(\theta_1) = \frac{3}{4}$, $u_o(\theta_2) = 1\frac{1}{4}$, $u_o(\theta_3) = 2\frac{1}{4}$.

(a) Explain why it is a Perfect Bayesian Equilibrium (PBE) outcome for type θ_1 worker to choose $q(\theta_1) = 0$ and the other types to choose their outside alternatives?

(b) What is the Intuitive Criterion? Does this PBE satisfy the Intuitive Criterion?

(c) Explain why it is a PBE for types θ_1 and θ_2 to signal with $q(\theta_1) = q(\theta_2) = 0$ and type θ_3 to choose $q(\theta_3) = 6$.

(d) Does the PBE satisfy the Intuitive Criterion?

(e) Show that there is a PBE that satisfies the Intuitive Criterion

(f) BONUS: Suppose henceforth that the outside payoffs are as follows: $u_o(\theta_1) = \frac{3}{4}$, $u_o(\theta_2) = 1\frac{3}{4}$, $u_o(\theta_3) = 2\frac{1}{2}$. Is there a PBE that satisfies the Intuitive Criterion? If not why not. If so, solve for the equilibrium signals and payoffs.

6. Monopoly and Product Quality

The set of buyer types is $\Theta = \{\theta_t\}_{t=1}^{T}$. If a type θ_t customer purchases a unit of quality level q and pays R, his benefit is $B(\theta_t, q) = \theta_t q$ and so his payoff is $u(\theta_t, q, R) = \theta_t q - R + u_o$.

No customer places any value on additional units. Let $\{q(\theta_t), R(\theta_t)\}_{\theta_t \in \Theta}$ be a set of quality levels and prices that satisfies the participation constraints.

(a) Show that a necessary condition for incentive compatibility is that $\{q(\theta_t)\}_{\theta_t \in \Theta}$ is increasing.

Henceforth consider the special three type case. Types are equally likely so that the probability of a type θ_t is $f(\theta_t) = \frac{1}{3}$. The cost of each unit of quality q is $C(q) = 2q^2$. The monopoly objective is expected profit maximization.

Consider the relaxed problem in which only the local downward incentive constraints are satisfied.

(b) Show that for profit maximization there are three binding constraints.

(c) Prove that the solution to the relaxed problem is incentive compatible for type θ_1.(The proof for other types is almost identical).

(d) Show that the expected profit of the firm can be written in the following form:

$$\Pi(q) = \frac{1}{3}\sum_{t=1}^{3}\left[A_t q(\theta_t) - 2q(\theta_t)^2 .\right]$$

[Confirm that $A_2 = 2\theta_2 - \theta_3$ and solve for A_1 and A_3].

(e) Hence obtain conditions under which only one quality level will be sold.