UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2013)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Revealed Preference

Suppose that you have (finite) data of price-consumption pairs $\mathcal{D} = \left\{ \left(p^t, x^t\right) \in \mathbb{R}^L_{++} \times \mathbb{R}^L_{+}, t = 1, ...T \right\}$ of some consumer. The following properties may or may not be satisfied by $\mathcal{D}$.

- **Weak Axiom of Revealed Preference (WARP)**: if x^t is revealed preferred to x^s (i.e. $p^t \cdot x^s \leq p^t \cdot x^t$) and $x^t \neq x^s$, then x^s is not revealed preferred to x^t.
- **Generalized Axiom of Revealed Preference (GARP)**: if x^t is indirectly revealed preferred to x^s (i.e. x^t is RPed to x^u and x^u is .....and x^v is RPed to x^s), then x^s is not strictly revealed preferred to x^t (i.e. $p^s \cdot x^t < p^s \cdot x^s$ does not hold).

We say that $\mathcal{D}$ is **rationalizable** if there exists a utility function $u : \mathbb{R}^L_+ \to \mathbb{R}$ such that $x^t \in \arg\max_{x \in \mathbb{R}^L_+} u(x)$ s.t. $p^t \cdot x \leq p^t \cdot x^t$ for $t = 1, ..., T$. Answer the following questions.

(a) Does GARP imply WARP? Either prove this or provide a counterexample.

(b) Does WARP imply GARP? Either prove this or provide a counterexample.

(c) Show that, if $\mathcal{D}$ can be rationalized by a locally nonsatiated utility function, then $\mathcal{D}$ must satisfy GARP.

(d) Show that, if $\mathcal{D}$ can be rationalized by a strictly quasi-concave utility function, then $\mathcal{D}$ must satisfy WARP.

2. Pareto Efficiency and Second Welfare Theorem

Consider a pure exchange economy $\mathcal{E}^{pure} = \left(\left\{\mathbb{R}_+^L, \succeq_i, e_i\right\}_{i=1,...,I}\right)$. Suppose that $\succeq_i$ is a continuous, locally nonsatiated and convex (and of course rational) preference on $\mathbb{R}_+^L$ for $i = 1, ..., I$.

(a) Define Pareto efficient allocation in pure exchange economy.

(b) Does there always exist a Pareto efficient allocation in such a pure exchange economy? Either prove it or provide a counterexample.

(c) Does there always exist a competitive equilibrium in such a pure exchange economy? Either prove it or provide a counterexample.

(d) For this question and the next question, suppose that every consumer's preference is identical, i.e. $\succeq_i = \succeq_j = \succeq$ for all i, j and that $\succeq$ satisfies monotonicity and the following stronger convexity property: if $x \succ y$, then $\alpha x + (1-\alpha)y \succ y$ for any $\alpha \in (0,1)$. Let $\overline{e} = \frac{\sum_{i=1}^{I} e_i}{I}$ be the average endowment. Show that $(\overline{e}, ..., \overline{e})$ is a Pareto efficient allocation.

(e) Suppose that every consumer's endowment is identical as well, i.e. $e_i = e_i = e$ and $e \gg 0$. Show that there exists a competitive equilibrium in $\mathcal{E}^{pure}$ where every consumer consumes e.(Hint: You should not apply the existence theorem (as we know) directly without strict convexity of preferences.)

3. A Crazy Bargaining Game

Two players $i \in \{1, 2\}$ play a war of attrition over continuous time $t \in [0, \infty)$. There is a pie of size 1 to be split between the two players. At the start of the game, each player i demands an exogenous share $\alpha \in (1/2, 1)$. Each player can then give into the other's demand at any time $t \geq 0$. If player j gives into i's demand at time t then they receive payoffs:

$$u_i = \alpha - kt \qquad \text{and} \qquad u_j = 1 - \alpha - kt$$

where $k > 0$ is an exogenous cost of bargaining.

(a) Player i's strategy can be described by a cdf $F_i(t)$ characterizing the time when they concede. Show that in any mixed strategy SPNE at most one agent concedes with positive probability at time $t = 0$ and after that each agent concedes with identical and constant hazard rates. Formally, there exist $c_1, c_2 \in [0, 1]$ with at least one of them equal to 1 such that their strategies are of the form

$$F_i(t) = 1 - c_i e^{-\lambda t}, \qquad t \in [0, \infty)$$

Is the equilibrium unique? [Hint: to derive F_i note that player i should be indifferent between conceding at time t and time $t + dt$.]

Now, suppose there is probability $z_i > 0$ that player i is crazy and never concedes. Assume wlog that $z_2 \geq z_1$.

(b) Let $F_i(t)$ represent the distribution of i's quitting times from j's perspective. Show that there exists $c_1 \in [0, 1]$, $c_2 = 1$ and T such that agent i's (sequential) equilibrium strategy is described by

$$F_i(t) = 1 - c_i e^{-\lambda t}, \qquad t \in (0, T]$$
$$F_i(t) = 1 - z_i, \qquad t \in (T, \infty)$$

Is the equilibrium unique?

(c) Fixing $z_2 > 0$, what happens to 1's strategy as $z_1 \to 0$? How do you interpret this?

4. Repeated Cournot & Bertrand

Consider an infinitely repeated oligopolistic market with two firms, demand function $p = 1 - Q$ and zero costs. First suppose that firms choose quantities as the strategic variable.

(a) Characterize Nash quantities and profits q^N, π^N and collusive quantities and profits q^C, π^C in the stage game.

(b) What is the minimum discount factor δ^* for which it is possible to sustain complete collusion, $q = q^C$, in SPNE using Nash reversion?

(c) Now assume that the firms are trying to collude on a different collusion level, $\hat{q}^C \in (q^C, q^N)$, in SPNE, using Nash reversion. Write down the necessary condition for δ! Is such collusion possible if $\delta < \delta^*$? Justify your answer qualitatively (or rigorously if you wish).

Next, suppose that firms choose prices as the strategic variable.

(d) Characterize Nash prices and profits p^N, π^N and cartel prices and profits p^C, π^C in the stage game.

(e) What is the minimum discount factor δ^* for which it is possible to sustain complete collusion, $p = p^C$, in SPNE, using Nash reversion?

(f) Now assume that the firms are trying to collude on a different collusion level, $\hat{p}^C \in (p^N, p^C)$, in SPNE, using Nash reversion. Write down the necessary condition for δ! Is such collusion possible if $\delta < \delta^*$? Justify your answer qualitatively (or rigorously if you wish).

5. Auctions

Each of n buyers wish to purchase a single unit of some commodity. Buyer i's value $\theta_i, i = 1, ..., n$ is continuously distributed on $[0, 1]$ with c.d.f. $F(\cdot)$. Initially assume that there is a single unit for sale.

(a) In an efficient auction the allocation is to a buyer with the highest value. Prove a buyer payoff equivalence theorem for all efficient mechanisms with a binding participation constraint. Then show that, from the seller's perspective, the expected payoff of each buyer is

$$\int_0^1 w(\theta)(1 - F(\theta))\, d\theta \text{ where } w(\theta) = F^{n-1}(\theta).$$

(b) Hence show that the expected revenue of the seller is

$$U^0 = n \int_0^1 w(\theta) J(\theta) F'(\theta)\, d\theta \text{ where } J(\theta) = \theta - \frac{1 - F(\theta)}{F'(\theta)}.$$

(c) Consider the sealed first and second price auctions with reserve price p. Prove a buyer payoff equivalence theorem and obtain an expression for expected seller revenue.

(d) What condition must be satisfied for the reserve price p to maximize expected revenue?

(e) Next suppose that there are two identical items for sale. Either prove the following statement is true or explain why it is false.

"With two identical items for sale it is still true that all efficient auctions with a binding participation constraint are buyer payoff (and hence revenue) equivalent."

6. Indirect Price Discrimination

A monopoly is prohibited from direct price discrimination but indirect price discrimination is legal. The unit cost of production is $c = 8$. There are three types of customer with demand price functions $p_1 = 20 - q_1$, $p_2 = 24 - q_2$, $p_3 = 32 - q_3$. The fraction of type t customers is $f_t, t = 1, 2, 3$.

Consider the plans $\{q_t, r_t\}_{t=1}^3$ where q_t is the number of units and r_t is total payment for these units.

(a) Prove that for these plans to be incentive compatible $\{q_t\}_{t=1}^3$ must be increasing.

(b) For any increasing $\{q_t\}_{t=1}^3$, show that for profit maximization the marginal informational rents are

$$U_2 - U_1 = 4q_1 \text{ and } U_3 - U_2 = 8q_2.$$

(c) Hence show that for revenue maximization

$$r_1 = 20q_1 - \frac{1}{2}q_1^2,\ r_2 = 24q_2 - \frac{1}{2}q_2^2 - 4q_1,\ r_3 = 32q_3 - \frac{1}{2}q_3^2 - 4q_1 - 8q_2.$$

(d) Write down an expression for expected profit and hence obtain expressions for the expected marginal profit for each type. Hence show that for some b, it is profit-maximizing to sell nothing to type 1 customers if and only if $f_1 \leq b$. (Remember that $f_1 + f_2 + f_3 = 1$.)

(e) Provide some intuition as to why an increase in f_2 and equal reduction in f_3 has no effect on the issue of whether it is profitable to sell to type 1 customers.

(f) What additional condition must be satisfied for it to be profit-maximizing to offer only a single plan?