UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2012)

Instructions: You have 4 hours for the exam. Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions. Use a SEPARATE bluebook to answer each question.

Problem 1 (Transfer and Destruction Paradoxes) The *Transfer Paradox* is the observation that (assuming outcomes come from a Walrasian Equilibrium), one agent may benefit (improve his/her equilibrium allocation) by transferring some of his/her endowment to some other agent before trade takes place. The *Destruction Paradox* is the observation that (assuming outcomes come from a Walrasian Equilibrium), one agent may benefit (improve his/her equilibrium allocation) by destroying some of his/her endowment before trade takes place. This problem asks you to show that the Destruction Paradox may occur in a setting where the Transfer Paradox does not.

Consider an exchange economy with two goods x, y and two agents. Endowments and utility functions of the two agents are

$$\begin{aligned} e_1 &= (A, 0) \quad , \quad U_1(x, y) &= \min\{\alpha x, y\} \\ e_2 &= (0, B) \quad , \quad U_2(x, y) &= \min\{x, \beta y\} \end{aligned}$$

where $A, B > 0$, $\alpha, \beta > 1$.

(a) For what values of the parameters α, β, A, B does this economy admit a Walrasian Equilibrium (WE) with strictly positive prices? For those values of the parameters, find the (unique) Walrasian Equilibrium allocation. (You do not have to find the WE price.)

(b) For those values of the parameters, show that Agent 1 cannot benefit by *transferring to* Agent 2 a small amount of his/her endowment before trade begins. That is, show that for small values of ε the Walrasian equilibrium from the initial endowments

$$e_1 = (A - \varepsilon, 0), e_2 = (\varepsilon, B)$$

is *not* better for Agent 1 than the Walrasian equilibrium in (b).

(c) Illustrate (a), (b) with an Edgeworth Box Diagram.

(d) For those values of the parameters, show that Agent 1 always benefits by *destroying* a small amount of of his/her endowment before trade begins. That is, show that for small values of ε the Walrasian equilibrium from the initial endowments

$$e_1 = (A - \varepsilon, 0), e_2 = (0, B)$$

is better for Agent 1 than the Walrasian equilibrium in (a).

Problem 2 (Betweenness) Let A_1, A_2, A_3 be alternatives, let $\mathcal{L}$ be the space of lotteries over these alternatives, and let $\succeq$ be a complete, transitive, reflexive, continuous preference relation on $\mathcal{L}$; assume $A_1 \succ A_2 \succ A_3$.

(a) Let $a, b \in \mathcal{L}$ and $p, q \in [0,1]$ with $p > q$. Show that there is some $\lambda \in (0,1)$ such that

$$\lambda[pa + (1-p)b] + (1-\lambda)b = qa + (1-q)b$$

(b) Assume that $\succeq$ also satisfies the following

$$\text{If } a, b \in \mathcal{L} \text{ with } a \sim b \text{ and } 0 \le p \le 1 \text{ then } a \sim pa + (1-p)b \sim b \quad (1)$$

Show that

$$\text{If } c, d \in \mathcal{L} \text{ with } c \succeq d \text{ and } 0 \le p \le 1 \text{ then } pc + (1-p)d \succeq d \quad (2)$$

Hint: If not then $d \succ pc + (1-p)d$; use the continuity and the result of part (a) for appropriate choices of a, b.

(c) If (1), (2) hold is it true that

$$\text{If } c, d \in \mathcal{L} \text{ with } c \succ d \text{ and } 0 < p < 1 \text{ then } ca + (1-p)d \succ d \quad (3)$$

For all parts, you may give algebraic arguments but it is acceptable – and may be easier – to give a diagrammatic argument using a Machina probability triangle.

3. Stackelberg Game with Noise

Consider the following extensive form game with player 1 ("leader") and player 2 ("follower"). First the leader chooses T ("Tough") or W ("Weak"). Then the follower observes the leader's action with noise. More specifically, the follower observes signal a $s \in \{t,w\}$ that is correct with probability $1-\varepsilon \in [0,1]$ as follows: $\Pr(s=t|a_1=T)=\Pr(s=w|a_1=W)=1-\varepsilon$ and $\Pr(s=w|a_1=T)=\Pr(s=t|a_1=W)=\varepsilon$. After observing a signal, the follower chooses F ("Fight") or R ("Retreat").

The payoff profile is (3, 0) if the leader chooses to be tough and the follower retreats, where the first entry is the leader's payoff and the second entry is the follower's payoff. The payoff profile is (5, 0) if the leader chooses W and the follower retreats. The other payoff profiles are (-1,-1) given (T, F) and (1, 1) given (W,F) respectively. Answer the following questions.

(a) Draw a game tree of this extensive form game.

(b) Consider a slightly different game in which the follower observes the leader's action directly instead of an imperfect signal. Find all subgame perfect equilibria in this game.

(c) Find all pure strategy sequential equilibria for this game with $\varepsilon \in (0, \frac{1}{4})$.

(d) Find all mixed strategy sequential equilibria in which the follower randomizes after observing w for this game with $\varepsilon \in (0, \frac{1}{4})$.

4. Bargaining with Random Proposer

Player α and β is playing a bargaining game to divide $1, where a role of proposer is assigned randomly in each period: each player is chosen as a proposer with probability (1/2) independently over time. The player chosen as a proposer makes an offer x∈ [0, 1], then the other player (responder) either accepts or rejects the offer. If the offer is accepted, then the responder receives x (and the proposer takes the rest 1-x) and the bargaining ends. The game moves on to the next period when the offer is rejected. Players discount payoffs that realize in period t by δ^{t-1}, where δ∈ (0, 1) is a common discount factor.

(a) Describe any one strategy in this dynamic bargaining game in detail. Make sure to describe all possible histories carefully.

(b) Find a stationary subgame perfect equilibrium (i.e. a proposer always makes the same offer and a responder always uses the same acceptance rule.

(c) Show that there exists the unique subgame perfect equilibrium in this game and derive the equilibrium payoff for a proposer and a responder in the beginning of the game after a role is assigned.

(d) Find any Nash equilibrium that is different from a subgame perfect equilibrium in (b) and (c)

5. Signaling

A type θ worker has a marginal product of $m(\theta) = \theta$. Types are uniformly distributed on [0,1] . There is no outside opportunity. A type θ can achieve education level q at a cost $C(\theta, q) = q / A(\theta)$ where $A(\theta)$ is positive and strictly increasing. Let $q(\theta)$ be the separating equilibrium education level of type θ . Firms do not observe θ but do observe q . Firms then play a Bertrand wage game

(a) Write down an expression for $U(\theta, x)$, the payoff to a type θ worker if he chooses education level $q(x)$.

(b) Let $V(\theta)$ be the equilibrium payoff of a type θ worker. Show that for equilibrium

(i) $q'(\theta) = A(\theta)$, (ii) $V'(\theta) = \dfrac{A'(\theta)}{A(\theta)} \dfrac{q(\theta)}{A(\theta)}$.

(c) Hence show that $A(\theta)V'(\theta) + A'(\theta)V(\theta) = \theta A'(\theta)$

(d) Solve for $V(\theta)$ (i) if $A(\theta) = \theta$, (ii) $A(\theta) = 2\theta$, (iii) $A(\theta) = \theta^2$. Hence rank the three signaling "technologies." Using a two type example, provide an explanation for this ranking.

(e) The solution appeals only to the FOC $\left.\dfrac{\partial U}{\partial x}(\theta, x)\right|_{x=\theta} = 0$. That is, incentive constraints hold locally. Show that all the incentive constraints are satisfied.

6. **Monopoly and product quality**

There are 3 types of consumer. A type *t* consumer is willing to pay $B_t(q) = a_t q^{1/2}$ for single unit of quality *q*, where $a_1 < a_2 < a_3$. There are n_t consumers of type t . The cost of producing each unit of quality *q* is $4q$.

(a) Let (q_t, r_t) be the choice of type $t = 1, ..., T$ (that is, a type *t* consumer pays r_t for quality level q_t) . Write down the optimization problem of a profit maximizing monopoly that cannot observe consumer preferences.

(b) Let (q_s, r_s) be the choice of type *s* and let (q, r) be another alternative offered in equilibrium by the monopoly. Show that higher types will never choose the alternative if $q < q_s$ and lower types will never choose the alternative if $q > q_s$. Hence the choices $\{q_t\}_{t=1}^{T}$ must be increasing.

(c) Provide a brief but clear explanation of why it is possible to ignore all but the monotonicity constraints and the "local downward constraints".

(d) If $(a_1, a_2, a_3) = (16, 24, 32)$ and there are equal numbers of each type, solve for the profit maximizing quality levels.