UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2010)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. First Welfare Theorem

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, \succeq_i, e_i\}_{i \in I})$ where $X_i = \mathbb{R}^L_+$, $\succeq_i$ is consumer i's preference and $e_i \in \mathbb{R}^L_+$ is consumer i's initial endowment.

(a) Define **Pareto efficient allocations** and **competitive (Walrasian) equilibrium** in this pure exchange economy.

(b) Prove that every competitive equilibrium allocation is Pareto efficient. State clearly any additional assumption you used to prove this claim.

2. Second Welfare Theorem

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, u_i, e_i\}_{i \in I})$ where $X_i = \mathbb{R}^L_+$, u_i is consumer i's continuous utility function on X_i and $e_i \in \mathbb{R}^L_+$ is consumer i's initial endowment. Suppose that $I = \{1\}$, i.e. there is only one consumer in this economy. Also suppose that 1's preference is strongly monotone, i.e. $u_1(x_1') > u_1(x_1'')$ if $x_1' \geq x_1''$ and $x_1' \neq x_1''$.

(a) There may not exist any competitive equilibrium in this economy. Illustrate this possibility by an example.

(b) Show that $e_1 \in \mathbb{R}^L_+$ can be supported by some competitive equilibrium when this economy satisfies a few more assumptions. Do not use the second welfare theorem. State clearly which additional assumption(s) you used.

3. Incomplete Markets

Consider an economy with two Consumers A, B, two dates $0, 1$, two states of nature $1, 2$ at date 1. A single consumption good is available at each date/state. Endowments and utility functions are

$$\begin{aligned} e^A &= (t; 1, 3) \\ u^A(x_0; x_1, x_2) &= \log x_0 + (1/2) \log x_1 + (1/2) \log x_2 \\ e^B &= (4 - t; 3, 1) \\ u^B(x_0; x_1, x_2) &= \log x_0 + (1/2) \log x_1 + (1/2) \log x_2 \end{aligned}$$

t is a parameter: $0 \leq t \leq 4$.

(a) Find the (unique) Walrasian equilibrium (prices and consumptions) for this economy. (The answer will involve the parameter t.)

Now suppose a single asset, paying the value of 1 unit of consumption in state 1 and 0 in state 2, is available for trade,

(b) Find the (unique) asset market equilibrium (prices and consumptions) for this economy. (The answer will involve the parameter t.)

(c) For what values of the parameter t (if any) does Consumer A prefer the incomplete asset market to the Walrasian market?

4. Jury voting with an absent-minded juror

Three jurors are deciding the fate of a person charged of murder. The person could be either guilty or innocent: $\omega \in \{G, I\}$, with a prior $\Pr(\omega = G) = \mu \in (1-p, 1/2)$. Jurors A and B are known to be open-minded and each have an assessment (or signal) of the person's guilt, $x_i \in \{G, I\}$, with $\Pr(x_i = \omega) = p > 1/2$, for $i = A, B$. Juror C on the other hand has slept through the entire trial proceedings and will randomize, and convict w.p. 1/2. The jurors equally dislike convicting an innocent and not convicting a guilty person (that is, they find it optimal to vote 'convict' if and only if their posterior that $\omega = G$, conditional on being the pivotal voter, exceeds 1/2).

(*i*) *Unanimity rule:* Suppose first that the Jury convicts only if there is unanimous agreement to convict. Show that there exists an equilibrium in which both open-minded jurors vote according to their signals.

Simple Majority Rule: Suppose next that the Jury decides according to a simple majority rule, and requires the agreement of two jurors to convict.

(*ii*) Does there exist a sincere voting equilibrium in which both open-minded jurors vote according to their signals?

(*iii*) Characterize an equilibrium of the voting game under simple majority rule. How does it compare to the equilibrium we computed in class, where all 3 jurors were open-minded?

(*iv*) Are the equilibria you characterized under (*i*) and (*iii*) unique, or do there exist other equilibria in the jury voting game?

5. Indirect Price Discrimination

A type θ agent has a production function $q = \theta L$, where L is hours worked. The set of types of agents is $\Theta = \{1, 2\}$ and there are equal numbers of each type. Preferences over output and labor are given by the utility function $U(q, L) = q - \frac{L^2}{14}$. Each agent has the option of emigrating in which case his utility is 1. The economy is ruled by a dictator interested in getting as much output as he can. Suppose first that he can identify types and so charge each a lump-sum tax.

(a) Show that a type agent's utility can be written as $q - \frac{1}{14}\left(\frac{q}{\theta}\right)^2 - t$.

(b) Solve for the optimal outputs of each type and the dictator's tax on each type.

Henceforth assume that an agent's type is private. The dictator thus offers the following tax mechanism

$$m = \{(q_1, t_1), (q_2, t_2)\}.$$

(c) Write down the constraints that must be satisfied for this mechanism to be incentive compatible and to satisfy participation constraints.

(d) Consider the "relaxed problem" where two of the constraints are dropped and explain why, in the solution to the relaxed problem, both constraints will be binding.

(e) Show that the single crossing property holds and hence explain briefly why it is possible to ignore the other 2 constraints.

(f) Solve for the dictator's optimal scheme.

6. Public Goods and Private Contributions

Each of 2 consumers have utility function $u_i(z, m_i) = z^{1/2} + m_i$. Note: the quantity z does not depend on i and is simultaneously available to both individuals.

The cost of z in terms of the money commodity is $c(z) = z$, i.e., if 1 and 2 contribute m_1 and m_2, the total amount of public good produced is $z = m_1 + m_2$.

(a) Assume that each individual takes the contribution of the other as given. Find the symmetric non-cooperative for this economy.

(b) Find the efficient level of production of the public good. Compare with (a) and explain your results.

(c) Suppose production of the public good is undertaken by a producer. A price-taking equilibrium for this model, also known as Lindahl equilibrium, is a $\bar{z}$ and p_1 and p_2, where p_i is the price to $i = 1, 2$, such that each consumer maximizes his utility and production is carried out under profit-maximization. Find the equilibrium and show that it is efficient.

(d) What would price-taking/Lindahl equilibrium be if $v_1(z) = \alpha z^{1/2}$ and $v_2(z) = (2-\alpha) z^{1/2}$ while c remained the same? What conclusions do you draw?

Price-taking equilibrium for private goods has been shown to be replica invariant, i.e., per capita gains are the same when the model is replicated. Moreover, the failure of replica invariance with private goods implies the non-existence of price-taking equilibrium.

(e) If instead of two (identical) individuals, there are four and the technology is the same, demonstrate that price-taking equilibrium of the kind exhibited in (c) exists for the four consumer model, but it is not replica invariant, Can you explain this in a way that shows it is not really an exception to replica invariance for private goods?