UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2009)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Expected Utility Theorem

Let Z be a finite set and $\triangle(Z)$ be the collection of lotteries on Z (i.e. each $p \in \triangle(Z)$ is a probability distribution on Z). Let $\succsim$ be some decision maker X's preference on $\triangle(Z)$. Assume that $\succsim$ is not a degenerate one: there exist $p, q \in \triangle(Z)$ such that $p \succ q$.

(a) When can X's preference $\succsim$ be represented by some utility function $U : \triangle(Z) \to \Re$? First define what "is represented by" means precisely, then list a collection of (nonredundant) axioms on $\succsim$ that guarantee such representation when they are satisfied. You do not have to prove the representation theorem.

(b) Expected utility theorem says that, when $\succsim$ satisfies certain axioms, there exists vNM utility function $u : Z \to \Re$ such that, for any $p, q \in \triangle(Z)$,

$$(*) \;\; p \succsim q \text{ if and only if } \sum_{z \in Z} p(z)\, u(z) \geq \sum_{z \in Z} q(z)\, u(z).$$

Suppose that u' is another vNM utility function u' that has the same property. Show that u' must be an affine transformation of u, i.e. there exist $\alpha > 0$ and $\beta \in \Re$ such that $u'(z) = \alpha u(z) + \beta$ for all $z \in Z$.

(c) Suppose that $\succsim$ can be represented by a continuous utility function $U : \triangle(Z) \to \Re$. It can be shown that $\succsim$ has a representation like $(*)$ when it satisfies one axiom. What is this axiom? State the axiom formally and prove this statement.

2. Pareto Efficiency

Consider a pure exchange economy $\mathcal{E}^{pure} = (\{X_i, u_i, e_i\}_{i \in I})$ where $X_i = \mathbb{R}^L_+$, $u_i : X_i \to \mathbb{R}$ is a continuous utility function of consumer i, and $e_i \in \mathbb{R}^L_+$ is consumer i's initial endowment.

(a) Define Pareto efficient allocations in pure exchange economy.

(b) Let $A \subset \mathbb{R}^{L \times I}_+$ be the set of feasible allocations in this economy. Show that, if $x \in \mathbb{R}^{L \times I}_+$ solves $\max_{x \in A} \sum_{i \in I} a_i u_i(x_i)$ for some $a = (a_1, ..., a_I) >> 0$, then x is a Pareto-efficient allocation.

(c) Consider a "converse" of (b). Suppose that $x \in \mathbb{R}^{L \times I}_+$ is a Pareto efficient allocation. Does x solves $\max_{x \in A} \sum_{i \in I} a_i u_i(x_i)$ for some $a = (a_1, ..., a_I) (\neq 0) \in \mathbb{R}^I_+$? Show that this is not correct without an additional assumption. Then find this additional assumption under which this statement is correct and prove this statement.

3. Problem (Game Theory)

There is one firm in a market and a potential entrant. The incumbent firm can have marginal cost of production c_L or c_H with probabilities q and $(1-q)$, and the entrant has marginal cost c, where $c_H > c > c_L$. The cost of the entrant is known by the incumbent. Furthermore, the entrant faces an entry cost $f > 0$. There are two periods, and in each period, there is a single consumer with known reservation value $p > c_H$. In the first period, the incumbent is alone and chooses a price p_1 at which he offers to sell a unit to the consumer. In the second period, after observing this price, the entrant decides whether to enter or not. After entry, it observes the cost of the incumbent. Both firms then choose simultaneously their prices, and the consumer chooses to purchase from the cheaper of the two (or the lower-cost firm, in case of a tie).

(a) Model this problem as a Bayesian game.

(b) Find all the separating ($p_H \neq p_L$) Perfect Bayesian equilibrium of this game.

(c) Characterize the pooling ($p_H = p_L$) equilibrium of this game. Does a pooling equilibrium always exist?

(d) Do all the pooling and separating equilibria you found in (b) and (c) satisfy the Intuitive Criterion?

4. Auctions

There are two buyers. Buyer i's valuation is an independent draw from a distribution with support $[0,1]$ and c.d.f. $F(v) \in C^1$. The seller announces a direct revelation mechanism with symmetric equilibrium allocation rule and expected payment $(\pi(v), r(v))$, where $\pi(v)$ is the probability a buyer is assigned the item if the value he submits is higher than the value announced by his opponent. The buyer who submits the lower value wins the item with zero probability. (In the case of a tie it does not matter who is assigned the item since, in equilibrium, this occurs with zero probability.)

(a) Briefly explain why $\pi(v)$ must be non-decreasing.

(b) Show that the equilibrium marginal informational rent is

$$\frac{dU_i}{dv_i} = \pi(v_i)F(v_i) \qquad (*)$$

Note: In part (f) you may wish to appeal to the fact that integrating by parts, it follows that the expected buyer payoff can be expressed as follows

$$\bar{U}_i = \int_0^1 U_i(v)F'(v)dv = \int_0^1 \pi(v)F(v)(1-F(v))dv + U_i(0).$$

(c) For that special case of the standard sealed high bid and sealed second bid auctions explain why (*) implies that the equilibrium buyer payoffs are the same in the two auctions.

(d) Does revenue equivalence also follow?

(e) Explain why the expected revenue of the seller is

$$\bar{U}_0 = 2\int_0^1 F(v)\pi(v)F'(v)dv - \bar{U}_1 - \bar{U}_2$$

(f) For the uniform case ($F(v) = v$), write down an expression for expected revenue and hence solve for the $\pi(v)$ that maximizes the payoff to the seller.

(g) How might this direct mechanism be implemented as an auction?

5. Classical and neo-classical value theory

There are two commodities, y and m, and two technologies:

$$Y_1 = \{(y_1, \beta_1) : y_1 \geq 0, 0 \geq c_1(y_1) \geq \beta_1\} \quad Y_2 = \{(y_2, \beta_2) : y_2 \leq 0, 0 \leq \beta_2 \leq c_2(y_2)\}.$$

In Y_1, $c_1(y_1) \leq 0$ is the minimum input of the m commodity needed to produce $y_1 \geq 0$ units of the other commodity while in Y_2, $c_2(y_2) \geq 0$ is the maximum amount of the m commodity that can be produced from $y_2 \leq 0$ units of the other commodity. Assume $c_1(0) = c_2(0) = 0$. Technologies are non-proprietary (e.g., they are blueprints available on the internet) that can be replicated indefinitely. Normalizing the price of the m commodity to be 1, the price of the other commodity is p.

(a) Write the profit function $\pi_j(p)$ for a single producer with technology Y_j, $j = 1, 2$. Letting $\Pi_j(p)$ be total profits for producers in industry j, explain why $\Pi(p) := \Pi_1(p) + \Pi_2(p) = 0$ is a *necessary* condition for p to be an equilibrium price vector.

There are three possibilities: (1) There is NO p satisfying $\Pi(p) = 0$; (2) There is exactly ONE p satisfying $\Pi(p) = 0$; (3) There is MORE than one p satisfying $\Pi(p) = 0$

(b) Illustrate each of these three possibilities in a diagram.

(c) Explain why (2) supports the Classical cost-of-production theory of value, i.e., tastes do not matter for price determination.

In Neo-classical theory, exchange economies are used to explain prices. There are two types of consumers and two commodities. Let (z_i, m_i) be the trades by type i, where $u_i(z_i, m_i) = v_i(z_i) + m_i$. Feasibility means $z_1 + z_2 = m_1 + m_2 = 0$. Let $A_i(z_i) = \{(y_i, \alpha_i) : v_i(y_i + z_i) + \alpha_i \geq v_i(z_i)\}$, $i = 1, 2$. In an infinitely replicated exchange economy, an arbitrager making deals involving utility non-decreasing trades with others would be able to obtain any trade in

$$A(z_1, z_2) = -[n_1 A_1(z_1) + n_2 A_2(z_2)],$$

where n_1, n_2 are any positive integers. (The arbitrageur is on the opposite side of the trade; hence, the minus sign.)

There are three possibilities: (I) $A(z_1, z_2)$ and $\mathbb{R}^2_{++}$ have a point in common; (II) there is ONE p such that $-A(z_1, z_2)$ is contained in the halfspace $H(p) \equiv \{(y, m) : py \geq m\}$; (III) there is MORE than one p for which $-A(z_1, z_2)$ is contained $H(p)$.

(d) Illustrate each of these three possibilities when v_i is concave.

(e) How do (I) and (II) compare with (1) and (2) in their implications for equilibrium?

6. Independent and interdependent investments

In period 1, individual $i = 1, 2$ makes an educational investment $e_i \geq 0$ at a cost of $c_i(e_i)$. Education influences the capacity to benefit from social programs $x \in X$ chosen in period 2. The payoff to i is $v_i(x, t_i(e_i))$ when the investment is e_i and the program is x, where $t_i(e_i)$ is, effectively, i's type after investing e_i.

(a) Assuming quasilinear utilities, define an expost efficient program when the types are (t_1, t_2).

(b) Assuming individuals report their types truthfully, define individual investment choices (e_1, e_2) that maximize overall efficiency.

(c) Using a Vickrey-Clarke-Groves scheme, show that there are money payments (m_1, m_2) in terms of reported types that would encourage individuals to undertake optimal investments in period 1 and truthfully report their types in period 2.

Suppose $t_1(e_1, e_2)$, i.e., 2's investment has a spillover effect on individual 1's capacity to benefit from social programs.

(d) Does this undermine the overall efficiency/truthtelling properties of the money payments scheme in (c)?

(e) If your answer to (d) is "yes," is there a modification that would deliver the desired properties?