# UCLA

## Department of Economics

## Ph. D. Preliminary Exam

## Micro-Economic Theory

(FALL 2008)

**Instructions**:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

## 1. Input and output prices in a $2 \times 2$ economy

An economy produces commodities $A$ and $B$ using inputs 1 and 2. The production function for each commodity is concave and homothetic. Commodity $A$ is more input 1 intensive than commodity $B$ for all input vectors. ( $\text{MRTS}_A(z) > \text{MRTS}_B(z)$). There is a fixed supply of each input.

(a) Sketch a proof that the output possibility set is convex.

(b) Use this result and an Edgeworth box diagram to explain why the input price ratio $r_1/r_2$ must rise as the output price ratio $p_A/p_B$ rises.

Henceforth consider two countries with identical technologies for producing two commodities but different input endowments that trade at the same world prices. Assume also that the production functions exhibit constant returns to scale.

(c) Explain why the input prices must be the same in both countries.

(d) Is it the case that if country 2 has twice the input endowment that it will produce twice the output of each commodity? Explain.

## 2. Equilibrium and time

Each consumer has the same homothetic utility function at dates $t = 1, 2$ given by

$$U(x) = \sum_{t=1}^{2} \sum_{j=1}^{2} \ln x_{tj}.$$

The aggregate endowment of commodity 1 is 20 in period 1 and 60 in period 2. Commodity 2 is produced from a single input oil that is in fixed supply. The production function in period 1 is $y_{12} = z_1^{1/2}/2$ and in period 2 is $y_{22} = z_2^{1/2}$. The total supply of the input is 800 ($z_1 + z_2 \leq 800$).

(a) Solve for the efficient outputs of commodity 2 in each period.

(b) Solve for the WE spot and futures prices of the outputs.

(c) Suppose that the spot price of commodity 1 is 1 in each period. What is the WE interest rate?

(d) Is there an equilibrium real interest rate in this economy? If so, what is it? If not, why not?

## 3. Bayesian Game

Batman has just learned that the Joker plans a big caper for tomorrow, but he does not know if the target of the Joker's attack will be the Museum or the Tower. Batman can guard one of these targets but not both. Tomorrow's results depend on the actions of Batman and the Joker and on the weather.

- If Batman guards the wrong target (that is, Batman does not guard the target the Joker attacks) then the Joker will succeed no matter what the weather. Batman values this outcome at $-4$ and the Joker values it as $+4$.
- If Batman guards the Museum and the Joker attacks the Museum and the weather is Bad, Batman will foil and catch the Joker. Batman values this outcome at $+8$ and the Joker values it at $-20$.
- In every other circumstance the Joker will be foiled but will escape. Batman and the Joker value this outcome at 0.

Batman must make his choice today – before he knows the weather. The Joker can make his choice tomorrow when he sees the weather. Neither player sees the action of the other. It is common knowledge that the probability the weather will be Good is 3/4 and the probability the weather will be Bad is 1/4.

Formalize this situation as a Bayesian Game and find all the Bayesian Nash equilibria.

## 4. Common Value Auction

2 bidders compete in a closed bid auction for a single indivisible item. The high bidder wins the item and pays the amount of his/her bid. Bidder's signals are drawn independently from the uniform distribution on $[0, 1]$; if signals are $s_1, s_2$ then the common value of the object is $s_1 s_2$.

Find the unique symmetric equilibrium in smooth strictly increasing bid functions.

## 5. Buying broadcast rights

PART I: Buyer $b$'s utility for broadcast channels $k = 1, \ldots, K$ is given by $v_b = \sum_k \beta_{bk} z_{bk}$, where $z_{bk} \in \{0, 1\}$, indicating that demand is indivisible and is limited to at most one unit of any channel. However, each $b = 1, \ldots, n$ may buy ANY number of channels. Buyers' tastes are quasi-linear and there is one unit of each channel available as supply.

(a) Formulate the problem defining an efficient allocation of channels to buyers.

(b) Describe a mechanism that provides incentives for buyers to bid truthfully and leads to an efficient allocation. Explain why your procedure works.

PART II: Valuations are as above, except that each buyer wants at most ONE channel.

(c) Formulate the problem defining an efficient allocation of channels to buyers.

(d) Describe a mechanism that provides incentives for buyers to bid truthfully and leads to an efficient allocation. Implement your procedure with respect to the following matrix $(\beta_{bk})$ where the rows are the buyers and the columns are the channels:

$$\begin{pmatrix} 7 & 4 \\ 5 & 6 \\ 6 & 4 \end{pmatrix}$$

## 6. Equilibrium with and without convex preferences

There are two commodities and two individuals, each with endowment $(1, 1)$. Their preferences are represented by utility functions of the form

$$U_\alpha = \alpha x^2 + y^2 \qquad \text{or} \qquad V_\beta = x^\beta y.$$

Prices are $p = (1, b)$, i.e., $b$ is the ratio of the price of $y$ to the price of $x$. Therefore, individual wealth is $w(p) = (1 + b)$.

(a) Draw a typical indifference curve for $U_\alpha$ and $V_\beta$. Find utility maximizing demand $D(b)$ as a function (or correspondence) of $b$ for an individual with utility $U_\alpha$.

(b) Show that there cannot be a price-taking equilibrium when $\alpha, \beta > 0$.

(c) Is there a price-taking equilibrium when there are three of type $\alpha = 4$ and one of type $\beta = 2$?

(d) Suppose a large number of individuals of type $U_\alpha$, where $\alpha = 1$, and an equally large number of $V_\beta$ with types that vary, say $1/3 \leq \beta \leq 3$. Find and explain why price-taking equilibrium could be said to exist, at least to within any desired degree of approximation.