UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2006)

Instructions:

- You have **4** hours for the exam
- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Walrasian Equilibrium and Time

Discuss each of the following statements. Make sure you take a clear position, even if you present arguments supporting or opposing a particular statement.

(a) "Adding time to the one-period N commodity Walrasian Equilibrium model is simply a matter of adding N markets for each of the T time periods. The welfare theorems trivially generalize."

(b) "All that is really needed is for the N commodity markets to remain open each period and to introduce $T - 1$ markets for bonds of each possible maturity date."

(c) "The T period economy generalizes directly to an infinite horizon Walrasian Equilibrium, simply by letting the number of periods increase without bound."

(d) "For the finite period case, even if consumers have very different beliefs, it is easy to incorporate uncertainty as well."

2. Monopoly response

A firm is a price setter in its output market but a price-taker in its m input markets. Suppose that the price of input 1 rises.

(a) In each case either show that the statement is true or explain why it is false. Note that you should understand any word like "rise" to mean "weakly rise."

(i) The firms demand for input 1 will fall.

(ii) The firms demand for the input will change more in the long-run than in the short run.

(iii) The firms output will fall.

(iv) Demand for the other inputs may rise or fall.

(b) How would you answer to (i) and (ii) change if the firm is also a price setter in the other input markets?

3. Subgame Confirmed Nash Equilibrium

Find all pure subgame confirmed Nash equilibria of the following three-player centipede game. Player 1 can drop out in which case payoffs are $(5,6,7)$. If he does not drop out, player 2 moves. If player 2 drops out payoffs are $(4,8,6)$. If player 2 does not drop out player 3 moves. If player 3 drops out payoffs are $(3,7,5)$. If player 3 does not drop out, all players get $(8,9,10)$.

4a Replicator

The replicator dynamic requires that the probabilities a strategy is used grow at a rate that is a linear function of the difference between the utility the strategy is getting and the mean payoff to any strategy. The continuous time best-response dynamic requires that the probabilities a strategy is used grow at a rate if the strategy is a best-response, and decline if it is not. In a 2x2 game with one population what is the relationship between a replicator and best-response dynamic?

5. Incentive-Efficiency Tradeoffs in Teams

Output y is produced by the joint efforts of two team members according to $y(e_1, e_2) = 3(e_1 e_2)^{1/3}$, $e_i \geq 0$. The utility to i is $U_i(y_i, e_i) = y_i - \sigma_i e_i$, where y_i is the quantity of y received by i and $\sigma_i > 0$ is the marginal disutility of effort.

(a) Find the efficient allocation of effort as a function of σ_1 and σ_2. [Suggestion: Compare the profit-maximizing choice of a price-taking producer facing a price of 1 for the output y and prices σ_1 and σ_2 for the inputs e_1 and e_2.]

[For parts (b) and (c), assume (σ_1, σ_2) is known and let $\bar{e} = (\bar{e}_1, \bar{e}_2)$ be the optimal solution to (a).]

(b) Suppose payments to team members equals output produced, i.e.,

$$(\dagger) \qquad y_1(e_1, e_2) + y_2(e_1, e_2) = y(e_1, e_2) \text{ for all } (e_1, e_2)$$

Using non-cooperative (Nash) equilibrium, show that there is NO reward scheme $y_1(e_1, e_2)$, $y_2(e_1, e_2)$ that would give team members the incentive to choose $\bar{e}$.

(c) Suppose one-sided balance, i.e., $(\dagger\dagger)$ $y_1(e_1, e_2) + y_2(e_1, e_2) \leq y(e_1, e_2)$, the difference being the penalty that team members pay to an outside mediator. Show that there IS a reward scheme (penalty function) for which $\bar{e}$ would be a non-cooperative equilibrium. Can you see possible problems with such a scheme?

(d) Is the reward scheme in (c) implementable if (σ_1, σ_2) is NOT known? Are there any reward schemes such that team members would have the incentive to reveal their σ_i so that the optimal $(\bar{e}_1(\sigma_1, \sigma_2), \bar{e}_2(\sigma_1, \sigma_2))$ could be implemented?

6. "Cournot" Monopolistic Competition with Large Numbers

In the economy with $N + 1$ individuals, the utility of i is

$$U_i^N(z_{i1}, \ldots, z_{iN+1}, m_i) = \sum_{j \neq i} v_N(z_{ij}) - c_N(z_{ii}) + m_i,$$

where $z_{ij} \geq 0$, $i \neq j$, and $z_{ii} \leq 0$, indicating that there are as many commodities as individuals, individual i is the only supplier of commodity $j = i$, and each individual does not consume the commodity he supplies (hence, there are N demanders). In addition,

$$v_N(z) = 10z - B_N z^2/2 \text{ and } c_N(z) = C_N z^2/2.$$

Therefore, the economy N is described by two numbers, $B_N > 0$ (the taste parameter) and $C_N > 0$ (the cost parameter). As consumers, individuals are price-takers, but as suppliers they can restrict the amount they supply (à la Cournot) by changing their cost functions from c_N to

$$c_N^K(z) = c_N(z) \text{ if } |z| \leq K, \quad c_N^K(z) = \infty, \text{ if } |z| > K, \text{ for any } K \geq 0.$$

Outcomes are determined by Walrasian equilibrium subject to the qualification that suppliers choose their capacity constraints (K) to maximize profits.

[The symmetry of the example implies that answers to the following questions can be obtained by examining demand and supply behavior in any one market.]

(a) Suppose $N = 1$ and $B_1 = C_1 = 1$. Show that it is profitable to impose capacity constraints.

(b) Under which of the situations below could individuals ignore capacity constraints and behave as simple price-takers as $N \to \infty$. Explain.

(i) $B_N = C_N = 1$ for all N

(ii) $B_N = N$, $C_N = 1$ for all N

(iii) $B_N = N$, $C_N = 1/N$

(iv) $B_N = 1$ for all N, $C_N = 1/N$