

UCLA
Department of Economics
Ph. D. Preliminary Exam
Micro-Economic Theory
(FALL 2004)

Instructions:

- You have 4 hours for the exam
- Answer any 5 out of the 6 questions. All questions are weighted equally. Answering fewer than 5 questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.
- Use a SEPARATE bluebook to answer each question.

1. Walrasian Equilibrium

- (a) Write down a simple model with the following features. Two commodities are sold in each of two periods. There is an endowment of each commodity in period 1 only. Commodity 1 can be costlessly stored. Period 2 output of commodity 2 is produced using inputs from period 1. The technology is subject to shocks. (S states). There are many consumers and firms. Each consumer has a utility function of the form $u^h(x) = U^h(x_{11}^h, x_{21}^h) + \delta U^h(x_{12}^h, x_{22}^h)$ where x_{jt}^h is consumer h 's consumption of commodity j in period t .
- (b) How many prices must there be for the market to be complete?
- (c) Characterize the Walrasian Complete Market Equilibrium of this economy.
- (d) Suppose markets will be open in both period 1 and period 2. If the future spot prices in each state are equal to the associated state claims prices are these Walrasian Equilibrium prices?
- (e) Suppose that in each period there is a spot market. In period 1 there is also a market for period 2 claims to commodity 2 in each state. However there is no state claims market for commodity 1. Under what assumptions (if any) would the Walrasian Equilibrium of your economy yield the complete market equilibrium outcome?
- (f) Suppose that there are no state claims markets but individuals can trade shares in a stock market. Under what conditions could this result in the complete market equilibrium outcome?

2. Cobb-Douglas Economy

- (a) A firm has a Cobb-Douglas production function $x = K^\alpha L^{1-\alpha}$, $0 < \alpha < 1$. What is the firm's cost function?
- (b) In a closed economy there are two commodities both produced using capital and labor according to Cobb-Douglas production functions. For commodity 1, $\alpha_1 = \frac{2}{3}$ and for commodity 2, $\alpha_2 = \frac{1}{3}$. Consumers have identical utility functions of the form

$$U(x_1, x_2) = 2 \ln x_1 + \ln x_2.$$

Units are normalized so that the aggregate supply of both capital and labor is 100.

Explain carefully why the optimal allocation of inputs is the solution of a “Robinson Crusoe” constrained optimization problem

(c) Solve for the optimal allocation of inputs. Hence solve for the equilibrium input price ratio and output price ratio.

(d) In the absence of an explicit utility function, what, if anything could be said about the input price ratio?

3. An Auction 3 bidders participate in a sealed-bid auction for a single indivisible item. Before the auction, each bidder i receives a signal s_i ; signals are drawn independently from the uniform distribution. Each bidder's true value is the sum of his/her own signal and the average of the other signals. (So bidder 1's value is

$$v_1 = s_1 + \frac{1}{2}(s_2 + s_3)$$

and so forth.) Derive carefully the unique symmetric Bayesian Nash equilibrium in smooth, strictly increasing strategies (bidding functions). [Assume bidders whose signals are 0 bid 0.]

4. Arbitration

John fell on the sidewalk outside Mary's Diner. Everyone agrees that the value of the injuries John sustained is \$100,000, but they do not agree whether Mary is liable for these injuries. John has therefore filed suit, and the trial is scheduled to begin tomorrow. If Mary is held liable, she will have to pay John \$100,000; if Mary is held not liable she will not have to pay John anything. The outcome of trial will depend on whether the jury believes Mary took proper care of her sidewalk. If Mary has a good record of care, she is strong and will win the case 80% of the time; if Mary has a bad record of care, she is weak and will win the case 20% of the time. If the case goes to trial, Mary will have to pay her lawyer \$10,000 whatever the outcome. (So if Mary wins the case, her payoff is -\$10,000 and John's payoff is \$0; if Mary loses the case, her payoff is -\$110,000 and John's payoff is \$100,000.)

However, John and Mary still have the opportunity to go to binding arbitration. If they do, the arbitrator will impose a settlement, which could include a payment (anything between \$0 and \$100,000) from Mary to John **and** a probability of going to trial (anything between 0 and 1) in addition to the settlement.

Mary knows whether she has a good record, but neither John nor the arbitrator know this. It is common knowledge that the probability that Mary has a good record is .5.

The following questions concern arbitration schemes that are incentive compatible and individually rational (both parties would be willing to participate, given all their information).

- a) Is there an arbitration scheme in which the arbitrator **never** sends the case to trial?
- b) For the Mary with a good record: What is the **worst expected outcome**?
- c) For the Mary with a good record: What is the **best expected outcome**?

5. Package Pricing

A collection of antique objects $A = \{1, \dots, n\}$ is to be sold. There is one unit of each. Assume for simplicity a single seller with zero reservation prices. Each buyer $b \in B = \{1, \dots, m\}$, with quasi-linear preferences, has a utility function described not only by reservation values for the individual objects, but also by reservation values for every package of objects $S \subseteq A$, denoted $v_b(S) \geq 0$. For example, the objects in a complete table setting of 18th century English china may be more highly valued than the sum of the objects separately; and, conversely, a table setting of 18th century china from England and a table setting of 18th century from the U.S. may be less highly valued together than the sum of their values regarded separately.

(a) Recognizing that objects and packages are indivisible, state the conditions for a *feasible* allocation of objects to buyers? For an *efficient* allocation? (Suggestion: Let S_b be the package of objects going to b .)

(b) Suppose the objects are priced individually (p_1, \dots, p_n) . What are the conditions for a *market-clearing equilibrium* among price-taking buyers and the single seller? If an object is not sold, what is its market-clearing price.

(c) Does the equilibrium in part (b) satisfies the efficiency condition in part (a)? Why? Why not?

(d) Pricing of individual objects may not always lead to equilibrium. Suppose two buyers and three objects. The buyers have identical tastes that are symmetric with respect to the number of objects: $v(\{i\}) = 0$, $i = 1, 2, 3$, $v(\{i, j\}) = 3$, $i \neq j$, and $v(\{1, 2, 3\}) = 4$. Show that for this example, there is no equilibrium of the type described in part (b).

(e) Suppose that prices exist for packages as well as for objects. Package prices need NOT have the *additivity* property, i.e., the price of the package need not equal the prices of the objects or the prices of the component packages in them. Find package prices such that there IS an equilibrium for the example in part (d) when buyers are price-takers with respect to package prices. Is it efficient?

6. Do profits measure marginal products?

There are 10 consumers. Each one has utility function

$$v(z) = 12z - 5z^2, \quad \frac{12}{10} \geq z \geq 0, [v(z) = -\infty \text{ otherwise}]$$

There are three producers. Each one has a cost function

$$c(z) = \frac{3z^2}{2}, \quad z \leq 0, \quad [c(z) = \infty \text{ otherwise}]$$

[Note: The convention is that supplies are negative.]

- (a) Find price-taking equilibrium.
- (b) Calculate the maximum gains from trade in this economy. Compare this to the gains in (a).
- (c) Compare, numerically, the gains consumers and producers receive in price-taking equilibrium with their marginal products. What is the source of the inequality?

Change the specification of producer costs in the above: there are two kinds of producers, high cost and low. High cost means

$$c(z) = -2z, \quad -6 \leq z \leq 0, \quad [c(z) = \infty \text{ otherwise}]$$

and low cost means

$$c(z) = -z, \quad -4 \leq z \leq 0, \quad [c(z) = \infty \text{ otherwise}]$$

There are two high cost and one low cost producers.

- (d) Repeat (a)–(c) for the economy with the revised production sector.
- (e) There is a tradition that profits are due to monopoly power and that firms make zero profits in perfectly competitive equilibrium. Can you reconcile that tradition with the profits for the low cost producer?