UCLA

Department of Economics

Ph. D. Preliminary Exam

Micro-Economic Theory

(FALL 2000)

Instructions:

- You have **4** hours for the exam

- Answer any **5** out of the **6** questions. All questions are weighted equally. Answering fewer than **5** questions is not advisable, so do not spend too much time on any question. Do NOT answer all questions.

- Use a SEPARATE bluebook to answer each question.

1. Radially Parallel Preferences

The preferences associated with a stricly increasing utility function $U(c)$ are radially parallel if, for any c and c' such that $U(c) = U(c')$, $U(\theta c) = U(\theta c')$ for all $\theta > 0$.

(a) Show that if $U(c) > U(c')$, then $U(\theta c) > U(\theta c')$ for all $\theta > 0$.

(b) Let $c(p, I)$ be the consumer's choice (assumed unique) when his income is I and prices are p. Show that $c(p, \theta I) = \theta c(p, I)$.

Suppose henceforth that $U(c) = \sum_{i=1}^{n} \alpha_i V(c_i)$, where $V'(c_i) = c^{-\sigma}$, $\sigma > 0$.

(c) Confirm that preferences are radially parallel.

(d) Suppose there are H consumers in an endowment economy, all with the same preferences. The aggregate supply of commodity i is ω_i, $i = 1, \ldots, n$. Solve for the equilibrium price ratios p_{i+1}/p_i in terms of the utilty parameters and aggregate endowments.

(e) Let $\alpha_i = \delta^{i-1}$, $i = 1, \ldots, n$, where $0 < \delta < 1$, and interpret the economy in (d) as one commodity economy with n periods. In addition, assume that $\omega_1 > \omega_2 > \cdots > \omega_n$. Show that for sufficiently low σ the one period interest rates are all positive and for sufficiently high σ they are all negative.

2. Short takes

In each case indicate whether the statement is true or false and then demonstrate your claim.

(a) A consumer must choose a portfolio made up of a riskless asset with a gross return of 1 and a risky asset which has a return of $1 + r_s$ is state s. If $\sum \pi_s r_s > 0$, where π_s is the probability of state s, the consumer will buy some of the risky asset even if he is highly risk averse.

(b) In an economy with no aggregate risk and risk averse consumers, the market value of an asset is the expected return to holding the asset, less a risk factor which depends on the degree of risk aversion.

(c) A multi-product firm which is a price-taker in its input markets has (weakly) downward sloping demands for its inputs, regardless of whether it is a price-taker or a monopolist in the output markets.

(d) In a 2×2 economy the production functions are $x_1 = K_1^{2/3} L_1^{1/3}$ and $x_2 = K_2^{1/2} L_2^{1/2}$. The aggregate supply of capital is $\overline{K}$ and the aggregate supply of labor is $\overline{L}$ and $\overline{L} = \overline{K}$. In such an economy the Walrasian equilibrium wage-rental ratio w/r is always in the internal $[1, 2]$.

3. Bayesian Games

Jules and Jim have stolen a shipment of bicycles, and have been caught. The prosecutor questions them separately, offering that if one testifies and the other does not, the testifier will receive 1 year in jail and the other will receive 6 years in jail, if both testify they will each get 3 years in jail, if neither testifies they will get 2 years in jail. [Caution: years in jail are BADS.] But there is a complication: the LA police are not very competent, and there is a chance that all the evidence will be thrown out on a technicality. If the evidence is thrown out and neither testifies, they will both go free; otherwise the outcomes will be the same as before.

Jules has a very smart lawyer and so *knows* whether or not the evidence will be thrown out, but Jim does not. However, it is common knowledge that similar cases are thrown out 2/3 of the time.

a) Model this story as a game of incomplete information, and find all the Bayesian Nash equilibria.

b) If Jules had the opportunity to communicate with Jim *after* Jules learns whether or not the evidence will be thrown out but *before* Jim is questioned, would the answer change? [Formal analysis not required.]

4. Repeated Competition

There are two providers of fishing guide services for tourists on Lake Tahoe. Because the number of catchable fish is limited, they have an incentive to cooperate [by limiting business], but they also have an incentive to cheat. In normal years [which occur 3/4 of the time], if the firms both cooperate they will each make a profit of $3 million; if one firm cheats and one cooperates, the cheating firm will make a profit of $5 million and the cooperator will make a profit of $0; if both firms cheat they will each make a profit of $1 million. But in bad market years [which occur 1/4 of the time], they will each make a profit of $0 no matter what they do. Firms can observe their own actions and payoffs but *cannot* observe the actions of the other firm or market conditions. Both firms discount the future at the constant discount rate $\delta = 2/3$

a) Model this story as a repeated game.

b) Find a symmetric perfect equilibrium in pure strategies that maximizes the social payoff.

5. Underproduction of Variety

The household sector is given by the tastes and endowment of a single aggregate consumer with utility function $U(x_1, x_2, \ldots) = \sum u(x_k)$ where

$$u(x_k) = \begin{cases} \sqrt{x_k} & \text{if } 0 \leq x_k \leq .5, \\ \sqrt{.5} + .1(x_k - .5) & \text{if } x_k > .5 \end{cases}$$

The cost of producing any commodity in terms of the labor input is given by

$$x_k = \begin{cases} 0 & \text{if } 0 \leq \ell_k \leq 1, \\ \sqrt{(\ell_k - 1)} & \text{if } \ell_k > 1 \end{cases}$$

and this technology is freely available. The size of the economy is measured by N, the number of units of the labor commodity ℓ with which the aggregate consumer is endowed. (Notice that ℓ does not appear in the utility function.)

(a) Show that beyond some point the demand curve facing a producer of any commodity is perfectly elastic.

(b) Assuming the price/unit of labor is unity, show that there is a monopolistically competitive equilibrium when the price for all produced commodities is 2, i.e., an equilibrium assuming the aggregate consumer is a price-taker for each produced commodity, each producing firm (identified with the single commodity it produces) maximizes profits subject to the demand for its product taking the price of labor as given, and each non-producing firm could not profitably undercut the price charged by a producing firm and could not profitably supply another commodity. (Suggestion: Ignore integer problems associated with N.)

(c) Show that in this monopolistically competitive equilibrium, for each produced commodity the same marginal conditions apply as in perfectly competitive equilibrium.

(d) Suppose $N = 200$. Find the number of commodities produced in monopolistically competitive equilibrium and show that there is another allocation of inputs to the production of commodities in which higher total utility for the aggregate consumer can be achieved by a 50% increase in the number of commodities produced.

(e) How can (d) be true if the marginal conditions for perfectly competitive equilibrium hold? Explain.

6. What happened to the importance property rights?

I. Steve Jobs, a co-founder of Apple Computers, has been quoted as saying that his biggest mistake was his strategy of making Apple computers and its operating system proprietary by scrupulously going after clone-makers and too slowly realizing that the specifications of its operating system should have been made accessible to anyone who wanted it. Gates, a co-founder of Microsoft, followed a different strategy. He encouraged the growth of clone makers through its sale of MS-DOS, even though many final users did not pay for it. This was the base upon which the dominance of Windows was established.

II. A Nobel laureate economist writing on the key to economic growth said:

> It is a unique feature of human capital [skills and knowledge] that it yields returns that cannot be captured entirely by its "owner." Bach and Mozart were well-paid (though neither as well as he thought he deserved!), but both of them provided enormous stimulation and inspiration to others, just as both of them also gained from others. Such *external effects*, as economists call them, are the subject matter of intellectual and artistic history, and should be the main subject of industrial and commercial history as well. There is thus a feedback effect in human capital theory: Something that increases the return on human capital will stimulate greater accumulation, in turn stimulating higher returns, stimulating still greater accumulation, and so on.

(a) Using standard principles of economic reasoning about the logic of property rights, explain why the opposite of I. and II. should be true, i.e., Jobs should be rich and Gates should be broke while economic growth should be slowed by external effects.

(b) Provide an explanation for why Jobs' strategy of enforcing property rights so that Apple could capture the gains from its innovation might very well have been very costly in terms of what Apple might have been able to appropriate had they followed Microsoft's strategy of not enforcing property rights. Does your explanation deny standard principles? Why, or why not?

(c) Provide an explanation for why growth might be favorably effected by the above kinds of externalities. Does your explanation distinguish between skills and knowledge? Does it deny standard principles, say with respect to patent laws?