Department of Economics **Econometrics Field Exam**
UCLA **Spring, 2010**

Second Year Field Examination in Econometrics
June 28, 2010

There is a total of nine questions, three in Part I, four in Part II, and two in Part III. Please answer six out of the nine available questions subject to the following restrictions:

1. You have to answer at least two questions from each of the first two parts, i.e., Part I and Part II.
2. Do not answer more than three questions from Part II of the exam.

Please provide as detailed information as possible. please keep in mind that the answers need not be long for them to be precise.

Part I:

Question 1:

Assume that, conditional on X_i, (Y_{1i}, Y_{0i}) is independent of D_i. We assume that $(Y_{1i}, Y_{0i}, D_i, X_i)$ is iid. Econometricians observe (Y_i, D_i, X_i) where $Y_i = D_i Y_{1i} + (1 - D_i) Y_{0i}$. We are interested in estimating the ATE

$$\beta = E[Y_{1i} - Y_{0i}] = E[\beta(X_i)]$$

where $\beta(x) = E[Y_{1i} - Y_{0i} | X_i = x]$.

Prove that

$$\beta = E\left[\frac{D_i Y_i}{p(X_i)} - \frac{(1 - D_i) Y_i}{1 - p(X_i)}\right]$$

where $p(x) = E[D_i | X_i = x]$.

Let

$$\widetilde{\beta} = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{D_i Y_i}{\widehat{p}(X_i)} - \frac{(1 - D_i) Y_i}{1 - \widehat{p}(X_i)}\right)$$

Sketch an argument why $\widetilde{\beta}$ should be consistent. Using the Newey formula, find the influence function of $\sqrt{n}\left(\widehat{\beta} - \beta\right)$.

Question 2:

Consider the linear regression model

$$y_{gi} = \alpha_g \beta + \varepsilon_{gi}, \quad i = 1, \ldots, n_g; g = 1, \ldots, G$$

where α_g denotes the unobserved group fixed effects. Here, the n_g denotes the size of the group g, and G denotes the number of groups.

We have a proxy for α_g from each individual:

$$x_{gi} = \alpha_g + v_{gi}$$

A reasonable procedure is to estimate α_g by

$$\overline{x}_g = \frac{1}{n_g} \sum_{i=1}^{n_g} x_{gi}$$

and regress y_{gi} on $\overline{x}_g$.

Letting

$$D_{gi}^{(k)} = \begin{cases} 1 & \text{if } g = k \\ 0 & \text{otherwise} \end{cases}$$

write

$$x_{gi} = \sum_{k=1}^{G} D_{gi}^{(k)} \alpha_g + v_{gi}$$

Prove that your OLS estimator is in fact a 2SLS. Provide a Nagar version of your estimator. You should be very detailed here. If you provide a generic formula for Nagar estimator, you will not get any credit for this portion of the question.

Question 3:

Consider the standard panel model

$$y_{it} = x_{it}\beta + \alpha_i + \varepsilon_{it}, \quad t = 1, 2$$

where we assume for simplicity that x_{it} is a scalar. Adopt the parametrization as in Mundlak:

$$\alpha_i = \overline{x}_i \pi + v_i$$

As we discussed in class, this parametrization suggests a control variable approach using $\overline{x}_i$. What is the implicit instrument?

Part II:

You may specify additional assumptions when necessary to provide an answer.

For questions 1, 2, and 3, consider the model

$$Y = m(X, \varepsilon)$$

where $Y \in R$ and $X \in R^K$ are observable, $\varepsilon \in R$ is unobservable, and m is strictly increasing in ε and continuous in (X, ε). Denote the cdf of ε by F_ε and the conditional distribution of Y given $X = x$ by $F_{Y|X=x}$.

Question 1:

Assume that ε is distributed independently of X with an everywhere positive density and that the support of X is R_+^K.

(a) Obtain an expression for $m(x, \varepsilon)$ in terms of $F_{Y|X}$ and F_ε.

(b) Let $y = m(x, \varepsilon)$. Obtain an expression for $m(x', \varepsilon) - m(x, \varepsilon)$ in terms of the distribution of the observable variables and y.

(c) Let $y = m(x, \varepsilon)$. Assume that all the functions are differentiable. Obtain an expression for $\partial m(x, \varepsilon) / \partial x$ in terms of the distribution of the observable variables and y.

(d) Show that if m is homogenous of degree one and its value is known at one point $(\overline{x}, \overline{\varepsilon})$, then F_ε and m can be identified in some set.

Question 2:

Suppose that X and ε are not independently distributed but there exists an observable W such that X is independent of ε conditional on W. Assume that the joint distribution of (Y, X, W) is observed.

(a) Explain how you can obtain in this case expressions in terms of the distribution of (Y, X, W) for (i) $m(x, \varepsilon)$, (ii) $m(x', \varepsilon) - m(x, \varepsilon)$, when $y = m(x, \varepsilon)$, and (iii) $\partial m(x, \varepsilon) / \partial x$ when $y = m(x, \varepsilon)$.

(b) Assume that m is homogenous of degree one and its value is known at one point $(\overline{x}, \overline{\varepsilon})$. Can you identify m and F_ε in this case? Explain.

Question 3:

Suppose that X and ε are not independently distributed, but it is known that for an observable Z that is independent of (ε, η) and some unknown function s that is strictly increasing in η

$$X = s(Z, \eta)$$

Assume that the joint distribution of (Y, X, Z) is observed.

(a) Explain how you can obtain expressions in terms of the distribution of (Y, X, Z) for (i) $m(x, \varepsilon)$, (ii) $m(x', \varepsilon) - m(x, \varepsilon)$, when $y = m(x, \varepsilon)$, and (iii) $\partial m(x, \varepsilon) / \partial x$ when $y = m(x, \varepsilon)$.

(b) Assume that m is homogenous of degree one and its value is known at one point $(\overline{x}, \overline{\varepsilon})$. Can you identify m and F_{ε} in this case? Explain.

Question 4:

Suppose that the model is

$$\begin{aligned} Y_1 &= m_1(Y_2, \varepsilon_1 + X_1) \\ Y_2 &= m_2(Y_1, \varepsilon_2 + X_2) \end{aligned}$$

where m_1 and m_2 are continuous functions, both strictly increasing in their last coordinates. Assume that (X_1, X_2) is distributed independently of $(\varepsilon_1, \varepsilon_2)$ and the distribution of (Y_1, Y_2, X_1, X_2) is observed. What can you say about the identification of the derivatives of m_1 and m_2? Provide details.

Part III:

Question 1—Dynamic Programming:

Consider a three-period model in which an individual makes decisions about three variables: consumption c_t, leisure l_t, and whether or not to attend school d_t, at each year, for $t = 1, 2, 3$. Note that while the first two variables are continuous choice variables, d_t is a binary choice variable that takes the value 1 if an individual decides to attend school at year t, and 0 otherwise.

Each period utility is given by

$$u_t(c_t, l_t) = u(c_t, l_t) = K_0 c_t^{\gamma_1} l_t^{\gamma_2},$$

Each individual gets a stream of unearned income. In each period that amount is given by a_t. If in addition the individual works then he/she is paid an hourly wage w_t, which is a function of the number of years of education denoted e, and experience, denoted x. Specifically, we have

$$\log w_t = \alpha_0 + \alpha_1 e_t + \alpha_2 e_t^2 + \alpha_3 x_t + \alpha_4 x_t^2 + \alpha_5 x_t e_t + \varepsilon_t,$$

where ε_t is an idiosyncractic shock, uncorrelated with either educator or experience.

The individual starts with an endowment of B_0, and earned an interest rate of r_t for any unused monetary resources carried over from period t to period $t + 1$. Assume that the interest rates for the three periods are known in advance.

For the questions specified below, if you think that some needed information has been omitted, please make assumption about that needed information.

1. Define the state vector, say z_t.
2. Specify all the necessary budget constraints, regarding money and time.
3. Write the value function at each period for the individual decision maker. Make sure to provide the value function for *all* possible state of nature and choices.
4. Provide the full list of parameters that need to be estimated. Let the true parameter vector be denoted by θ_0.
5. Provide details about the data that one would need in order to be able to estimate θ_0.
6. Provide detailed information about the method by which you propose to estimate the parameter vector θ_0. You should answer this question providing very detailed information, assuming you are giving instructions to a professional programmer who knows nothing about economics. (Keep in mind that there are several alternative ways for obtaining an estimate for θ_0.)
7. What is the asymptotic distribution for the parameter vector estimate, say $\widehat{\theta}_n$, obtained following the procedure suggested in part (6)? Provide brief justifications for all your claims.

Question 2—Generalized Method of Moments:

Consider the two moment functions given by $\varphi_1(W_i, \theta_1)$ and $\varphi_2(W_i, \theta_1, \theta_2)$. Suppose that, when evaluated at the true population parameter vectors, θ_{01} and θ_{02}, respectively, we have

$$\begin{aligned} E[\varphi_1(W_i, \theta_{01})] &= 0 \quad \text{and} \\ E[\varphi_2(W_i, \theta_{01}, \theta_{02})] &= 0, \end{aligned}$$

where W_i, $i = 1, ..., n$, represent cross-section independent and identically distributed (i.i.d.) data.

Let $\varphi_1(\cdot)$ and $\varphi_2(\cdot)$ be $M_1 \times 1$ and $M_2 \times 1$ vector-valued functions, respectively, and correspondingly let θ_{01} and θ_{02} be d_1 and d_2 vectors of parameters, with $M_1 > d_1$ and $M_2 = d_2$.

Assume that you have already obtained an estimator for θ_{01}, say $\widehat{\theta}_{n1}$, and this estimator is consistent and asymptotically normal, i.e.,

$$\widehat{\theta}_{n1} \xrightarrow{p} \theta_{01},$$
$$\sqrt{n}\left(\widehat{\theta}_{n1} - \theta_{01}\right) \xrightarrow{D} N\left(0, V_1\right),$$

as $n \to \infty$.

1. Suggest an efficient Generalized Method of Moments (GMM) estimator for θ_{02}, using a plug-in estimator for θ_{01}, namely $\widehat{\theta}_{n1}$.

2. Show that the estimator suggested in (1) is consistent and has an asymptotic normal distribution. In doing so explicitly state all the assumptions you make. Provide the asymptotic properties of the estimator $\widehat{\theta}_{n2}$ for θ_{02}. Justify each and every step in your answer.

3. Suppose now that parameter vector θ_{01} is known, so that in order to obtain an estimator for θ_{02} we can use θ_{01} rather than an estimator for it. How are the properties of the estimator obtained here different from those of the estimator obtained in (2)? Justify your answer.

4. One suggested that instead of following a two-step procedure as described above one can obtain estimators for θ_{01} and θ_{02} by simply solving for $\widehat{\theta}_{n1}$ and $\widehat{\theta}_{n2}$ simultaneously. Offer your detailed opinion regarding such a proposal.