Second Year Field Examination in Econometrics  
June 28, 2010

There is a total of nine questions, three in Part I, four in Part II, and two in Part III. Please answer six out of the nine available questions subject to the following restrictions:

1. You have to answer at least two questions from each of the first two parts, i.e., Part I and Part II.

2. Do not answer more than three questions from Part II of the exam.

Please provide as detailed information as possible. Please keep in mind that the answers need not be long for them to be precise.

Part I:

Question 1:

Assume that, conditional on \(X_i\), \((Y_{1i}, Y_{0i})\) is independent of \(D_i\). We assume that \((Y_{1i}, Y_{0i}, D_i, X_i)\) is iid. Econometricians observe \((Y_i, D_i, X_i)\) where \(Y_i = D_iY_{1i} + (1 - D_i)Y_{0i}\). We are interested in estimating the ATE

\[
\beta = E[Y_{1i} - Y_{0i}] = E[\beta(X_i)]
\]

where \(\beta(x) = E[Y_{1i} - Y_{0i} \mid X_i = x]\).

Prove that

\[
\beta = E\left[\frac{D_iY_i}{p(X_i)} - \frac{(1 - D_i)Y_i}{1 - p(X_i)}\right]
\]

where \(p(x) = E[D_i \mid X_i = x]\).

Let

\[
\tilde{\beta} = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{D_iY_i}{\tilde{p}(X_i)} - \frac{(1 - D_i)Y_i}{1 - \tilde{p}(X_i)} \right)
\]

Sketch an argument why \(\tilde{\beta}\) should be consistent. Using the Newey formula, find the influence function of \(\sqrt{n}(\tilde{\beta} - \beta)\).

Question 2:

Consider the linear regression model

\[
y_{gi} = \alpha_g\beta + \varepsilon_{gi}, \quad i = 1, \ldots, n_g; g = 1, \ldots, G
\]

where \(\alpha_g\) denotes the unobserved group fixed effects. Here, the \(n_g\) denotes the size of the group \(g\), and \(G\) denotes the number of groups.

We have a proxy for \(\alpha_g\) from each individual:

\[
x_{gi} = \alpha_g + v_{gi}
\]
A reasonable procedure is to estimate $\alpha_g$ by

$$\bar{x}_g = \frac{1}{n_g} \sum_{i=1}^{n_g} x_{gi}$$

and regress $y_{gi}$ on $\bar{x}_g$.

Letting

$$D_g^{(k)} = \begin{cases} 
1 & \text{if } g = k \\
0 & \text{otherwise}
\end{cases}$$

write

$$x_{gi} = \sum_{k=1}^{G} D_g^{(k)} \alpha_g + v_{gi}$$

Prove that your OLS estimator is in fact a 2SLS. Provide a Nagar version of your estimator. You should be very detailed here. If you provide a generic formula for Nagar estimator, you will not get any credit for this portion of the question.

**Question 3:**

Consider the standard panel model

$$y_{it} = x_{it} \beta + \alpha_i + \epsilon_{it}, \quad t = 1, 2$$

where we assume for simplicity that $x_{it}$ is a scalar. Adopt the parametrization as in Mundlak:

$$\alpha_i = \bar{x}_i \pi + v_i$$

As we discussed in class, this parametrization suggests a control variable approach using $\bar{x}_i$. What is the implicit instrument?
Part II:

You may specify additional assumptions when necessary to provide an answer.

For questions 1, 2, and 3, consider the model

\[ Y = m(X, \varepsilon) \]

where \( Y \in \mathbb{R} \) and \( X \in \mathbb{R}^K \) are observable, \( \varepsilon \in \mathbb{R} \) is unobservable, and \( m \) is strictly increasing in \( \varepsilon \) and continuous in \((X, \varepsilon)\). Denote the cdf of \( \varepsilon \) by \( F_\varepsilon \) and the conditional distribution of \( Y \) given \( X = x \) by \( F_{Y|x=x} \).

Question 1:

Assume that \( \varepsilon \) is distributed independently of \( X \) with an everywhere positive density and that the support of \( X \) is \( \mathbb{R}^K_+ \).

(a) Obtain an expression for \( m(x, \varepsilon) \) in terms of \( F_{Y|X=x} \) and \( F_\varepsilon \).

(b) Let \( y = m(x, \varepsilon) \). Obtain an expression for \( m(x', \varepsilon) - m(x, \varepsilon) \) in terms of the distribution of the observable variables and \( y \).

(c) Let \( y = m(x, \varepsilon) \). Assume that all the functions are differentiable. Obtain an expression for \( \partial m(x, \varepsilon) / \partial x \) in terms of the distribution of the observable variables and \( y \).

(d) Show that if \( m \) is homogenous of degree one and its value is known at one point \((x, \varepsilon)\), then \( F_\varepsilon \) and \( m \) can be identified in some set.

Question 2:

Suppose that \( X \) and \( \varepsilon \) are not independently distributed but there exists an observable \( W \) such that \( X \) is independent of \( \varepsilon \) conditional on \( W \). Assume that the joint distribution of \((Y, X, W)\) is observed.

(a) Explain how you can obtain in this case expressions in terms of the distribution of \((Y, X, W)\) for (i) \( m(x, \varepsilon) \), (ii) \( m(x', \varepsilon) - m(x, \varepsilon) \), when \( y = m(x, \varepsilon) \), and (iii) \( \partial m(x, \varepsilon) / \partial x \) when \( y = m(x, \varepsilon) \).

(b) Assume that \( m \) is homogenous of degree one and its value is known at one point \((x, \varepsilon)\). Can you identify \( m \) and \( F_\varepsilon \) in this case? Explain.

Question 3:

Suppose that \( X \) and \( \varepsilon \) are not independently distributed, but it is known that for an observable \( Z \) that is independent of \((\varepsilon, \eta)\) and some unknown function \( s \) that is strictly increasing in \( \eta \)

\[ X = s(Z, \eta) \]

Assume that the joint distribution of \((Y, X, Z)\) is observed.

(a) Explain how you can obtain expressions in terms of the distribution of \((Y, X, Z)\) for (i) \( m(x, \varepsilon) \), (ii) \( m(x', \varepsilon) - m(x, \varepsilon) \), when \( y = m(x, \varepsilon) \), and (iii) \( \partial m(x, \varepsilon) / \partial x \) when \( y = m(x, \varepsilon) \).
(b) Assume that $m$ is homogenous of degree one and its value is known at one point $(\bar{x}, \bar{y})$. Can you identify $m$ and $F_\varepsilon$ in this case? Explain.

**Question 4:**

Suppose that the model is

\[
Y_1 = m_1(Y_2, \varepsilon_1 + X_1) \\
Y_2 = m_2(Y_1, \varepsilon_2 + X_2)
\]

where $m_1$ and $m_2$ are continuous functions, both strictly increasing in their last coordinates. Assume that $(X_1, X_2)$ is distributed independently of $(\varepsilon_1, \varepsilon_2)$ and the distribution of $(Y_1, Y_2, X_1, X_2)$ is observed. What can you say about the identification of the derivatives of $m_1$ and $m_2$? Provide details.
Part III:

Question 1—Dynamic Programming:

Consider a three-period model in which an individual makes decisions about three variables: consumption \( c_t \), leisure \( l_t \), and whether or not to attend school \( d_t \), at each year, for \( t = 1, 2, 3 \). Note that while the first two variables are continuous choice variables, \( d_t \) is a binary choice variable that takes the value 1 if an individual decides to attend school at year \( t \), and 0 otherwise.

Each period utility is given by

\[
U_t(c_t, l_t) = K_0 c_t^\gamma_1 l_t^\gamma_2,
\]

Each individual gets a stream of unearned income. In each period that amount is given by \( a_t \). If in addition the individual works then he/she is paid an hourly wage \( w_t \), which is a function of the number of years of education denoted \( e \), and experience, denoted \( x \). Specifically, we have

\[
\log w_t = \alpha_0 + \alpha_1 e_t + \alpha_2 e_t^2 + \alpha_3 x_t + \alpha_4 x_t^2 + \alpha_5 x_t e_t + \varepsilon_t,
\]

where \( \varepsilon_t \) is an idiosyncratic shock, uncorrelated with either educator or experience.

The individual starts with an endowment of \( B_0 \), and earned an interest rate of \( r_t \) for any unused monetary resources carried over from period \( t \) to period \( t + 1 \). Assume that the interest rates for the three periods are known in advance.

For the questions specified below, if you think that some needed information has been omitted, please make assumption about that needed information.

1. Define the state vector, say \( z_t \).
2. Specify all the necessary budget constraints, regarding money and time.
3. Write the value function at each period for the individual decision maker. Make sure to provide the value function for all possible state of nature and choices.
4. Provide the full list of parameters that need to be estimated. Let the true parameter vector be denoted by \( \theta_0 \).
5. Provide details about the data that one would need in order to be able to estimate \( \theta_0 \).
6. Provide detailed information about the method by which you propose to estimate the parameter vector \( \theta_0 \). You should answer this question providing very detailed information, assuming you are giving instructions to a professional programmer who knows nothing about economics. (Keep in mind that there are several alternative ways for obtaining an estimate for \( \theta_0 \).)
7. What is the asymptotic distribution for the parameter vector estimate, say \( \hat{\theta}_n \), obtained following the procedure suggested in part (6)? Provide brief justifications for all your claims.

Question 2—Generalized Method of Moments:

Consider the two moment functions given by \( \varphi_1(W_i, \theta_1) \) and \( \varphi_2(W_i, \theta_1, \theta_2) \). Suppose that, when evaluated at the true population parameter vectors, \( \theta_{01} \) and \( \theta_{02} \), respectively, we have

\[
E[\varphi_1(W_i, \theta_{01})] = 0 \quad \text{and} \quad E[\varphi_2(W_i, \theta_{01}, \theta_{02})] = 0,
\]

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where \( W_i, i = 1, ..., n \), represent cross-section independent and identically distributed (i.i.d.) data.

Let \( \varphi_1(\cdot) \) and \( \varphi_2(\cdot) \) be \( M_1 \times 1 \) and \( M_2 \times 1 \) vector-valued functions, respectively, and correspondingly let \( \theta_{01} \) and \( \theta_{02} \) be \( d_1 \) and \( d_2 \) vectors of parameters, with \( M_1 > d_1 \) and \( M_2 = d_2 \).

Assume that you have already obtained an estimator for \( \theta_{01} \), say \( \hat{\theta}_{n1} \), and this estimator is consistent and asymptotically normal, i.e.,

\[
\hat{\theta}_{n1} \xrightarrow{p} \theta_{01}, \quad \sqrt{n} \left( \hat{\theta}_{n1} - \theta_{01} \right) \xrightarrow{D} N(0, V_1),
\]

as \( n \to \infty \).

1. Suggest an efficient Generalized Method of Moments (GMM) estimator for \( \theta_{02} \), using a plug-in estimator for \( \theta_{01} \), namely \( \hat{\theta}_{n1} \).

2. Show that the estimator suggested in (1) is consistent and has an asymptotic normal distribution. In doing so explicitly state all the assumptions you make. Provide the asymptotic properties of the estimator \( \hat{\theta}_{n2} \) for \( \theta_{02} \). Justify each and every step in your answer.

3. Suppose now that parameter vector \( \theta_{01} \) is known, so that in order to obtain an estimator for \( \theta_{02} \) we can use \( \theta_{01} \) rather than an estimator for it. How are the properties of the estimator obtained here different from those of the estimator obtained in (2)? Justify your answer.

4. One suggested that instead of following a two-step procedure as described above one can obtain estimators for \( \theta_{01} \) and \( \theta_{02} \) by simply solving for \( \hat{\theta}_{n1} \) and \( \hat{\theta}_{n2} \) simultaneously. Offer your detailed opinion regarding such a proposal.