UCLA Department of Economics

Field Comprehensive Examination in ECONOMETRICS

Fall 2002

Answer ALL questions in Parts I, II, and III
Use a separate answer book for each Part.

Each part has equal weight.
You have four hours to complete the exam.
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Econometrics Field Exam

PART I:

Suppose you have \( n \) i.i.d. observations on \((y_i, X_i)\) from the following discrete choice model:

\[
y_i = \begin{cases} 
1 & \text{if } X_i\beta + \epsilon_i \geq 0 \\
0 & \text{if } X_i\beta + \epsilon_i \leq 0
\end{cases}
\]

where \( X_i \) is a \((1 \times k)\) random vector that contains a constant and \( \epsilon_i \) is an unobservable error term has a standard normal distribution with mean 0 and variance equal to 1.

(a) Find the mean and variance of \( y_i \) given \( X_i \).

Describe the following estimators of \( \beta \) and derive their asymptotic distribution:

(b) The non-linear least squares estimator.

(c) The weighted non-linear least squares estimator.

(d) The maximum likelihood estimator.

(e) The efficient GMM estimator that uses all moment conditions implied by the model.

(f) The empirical likelihood estimator.
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PART II

1. Consider the panel data model

\[ y_{it} = \alpha_i + \beta' x_{it} + u_{it} \quad t = 1, 2 : i = 1, \ldots, n, \]

where it is known that

\[ E[u_{it} | \alpha_i, x_{i1}, x_{i2}] = 0 \quad t = 1, 2. \]

For each \( i \), we observe \( z_i \equiv (y_{i1}, y_{i2}, x_{i1}, x_{i2}) \).

(a) Show that the fixed effects estimator is numerically identical to the OLS estimator based on the first difference

\[ y_{i2} - y_{i1} = \beta' (x_{i2} - x_{i1}) + u_{i2} - u_{i1} \]

(b) Derive the asymptotic distribution of the fixed effects estimator as \( n \to \infty \).

(c) Describe a condition under which the between estimator based on

\[ \frac{y_{i2} + y_{i1}}{2} = \beta' \left( \frac{x_{i2} + x_{i1}}{2} \right) + \frac{u_{i2} + u_{i1}}{2} \]

consistent as \( n \to \infty \).

2. Consider the panel data model

\[ y_{it} = \alpha_i + \beta \cdot x^*_{it} + u_{it} \quad t = 1, 2, 3 : i = 1, \ldots, n, \]

where it is known that

\[ E[u_{it} | \alpha_i, x^*_{i1}, x^*_{i2}, x^*_{i3}] = 0 \quad t = 1, 2, 3. \]

For simplicity, it is assumed that \( \text{dim} (\beta) = 1 \). For each \( i \), we observe \( z_i \equiv (y_{i1}, y_{i2}, y_{i3}, x_{i1}, x_{i2}, x_{i3}) \), where

\[ x_{it} = x^*_{it} + \epsilon_{it} \]

Here \( (\epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3}) \) is the measurement error independent of \( (y_{i1}, y_{i2}, y_{i3}, x_{i1}^*, x_{i2}^*, x_{i3}^*, u_{i1}, u_{i2}, u_{i3}, \alpha_i) \).

We also assume that the measurement errors are independent of each other, i.e., \( \epsilon_{it} \) is independent of \( \epsilon_{t'} \) for \( t \neq t' \).

(a) Show that the OLS estimator based on the first difference

\[ y_{i2} - y_{i1} = \beta \cdot (x_{i2} - x_{i1}) + u_{i2} - u_{i1} \]

is inconsistent as \( n \to \infty \).

(b) Consider using \( x_{i3} \) as an instrument to the first difference

\[ y_{i2} - y_{i1} = \beta \cdot (x_{i2} - x_{i1}) + u_{i2} - u_{i1} \]

Show that the resultant estimator is consistent as \( n \to \infty \).
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PART III

Suppose one observed the following autocorrelations for lags 1 to 4:

.43 .08 .04 -.05

Given these autocorrelations, how would you test whether the series was white noise? (There are 100 observations)

What type of stochastic process do you think generated these autocorrelations?

Suppose that the innovations to the process are Gaussian. Write down the conditional log-likelihood, and describe how would you estimate the parameters of this stochastic process. (Make any assumptions you need).

Suppose you want to estimate the parameters of the following process:

\[ y_t = \mu + \rho y_{t-1} + \varepsilon_t, \varepsilon \sim N(0, \sigma^2) \]

Describe why OLS is equivalent to conditional MLE.

Suppose the least squares estimate of \( \rho = .97 \), and there are 100 observations. Do you think the process is stationary? If not, how would you test for stationarity?

Describe what the concept of cointegration means. Suppose you found that for the United States that real GDP and real consumption were not cointegrated. What would you conclude from this finding?