UCLA Department of Economics **Spring 2014**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, and you are to complete each part. **Answer each question in a separate bluebook.** All three parts will receive equal weight in your grade.

Part 1

Consider a standard neoclassical growth model where a single good is produced according to a Cobb-Douglas production function from capital and labor: $y_t = e^{z_t} k_t^{\theta} h_t^{1-\theta}$. Here, z_t is a random variable revealed at the beginning of period *t*. Output can be consumed (c_t) or invested (x_t), where $k_{t+1} = (1-\delta)k_t + x_t$.

A. The following are possible specifications for preferences:

(i) $$E\sum_{t=0}^{\infty} \beta^t \{\log c_t - A\frac{h_t^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\},\ A > 0,\ \omega > 0$$

(ii) $$E\sum_{t=0}^{\infty} \beta^t \frac{\left(c_t - A\frac{h_t^{1+\frac{1}{\omega}}}{1+\frac{1}{\omega}}\right)^{1-\sigma}}{1-\sigma},\ \omega > 0, A > 0, \sigma > 1.$$

What does it mean to say that preferences are consistent with balanced growth? In each case, is the utility function consistent with balanced growth? Prove your claims formally (don't just quote a theorem).

B. Decentralize this economy and define a *recursive competitive equilibrium*. Assume markets for output, labor, capital services, and privately issued one-period bonds.

C. The *Frisch elasticity of labor supply* is defined to be the elasticity of hours worked to the wage rate holding the marginal utility of wealth (the Lagrange multiplier for the budget constraint) constant. Derive the Frisch elasticity for your decentralized economy for each of the two utility functions.

D. If the utility function is given by (ii) above, would your expression for the labor supply elasticity change if you did not assume that the marginal utility of wealth were constant? Explain.

E. Assuming preference specification (i) in part A, derive an expression from the first order condition for hours worked that relates consumption, the wage rate and hours worked in equilibrium. Log-linearize this equation to express the (approximate) relationship between the percentage deviation of these variables from steady state. What does this linear relationship imply about the standard deviation of these variables? What does the model described in this question imply about the standard deviation of wages relative to the standard deviation of average labor productivity? Explain.

F. A typical empirical estimate of the Frisch elasticity from micro data is about 0.5. Given your findings from part E, is this value consistent with the business cycle statistics computed from aggregate U.S. data (use average labor productivity to proxy for the wage rate)? Explain.

G. Describe one way (several exist in the literature) that the model described in this question can be modified in order to reconcile large fluctuations in hours relative to labor productivity at the aggregate level with low micro estimates of the labor supply elasticity.

Question 2
June 2014

1 Factor Shares and Inequality in two growth models

Consider an overlapping generations economy in which time is denoted $t = 1, 2, 3, \ldots$. Each period t, a new cohort of agents is born, is young in period t, and old in period $t+1$. We assume that each agent in that cohort is endowed with 1 unit of labor while young and nothing else. We denote the consumption of agents in this cohort by c_t^y, c_{t+1}^o and they have utility $\log(c_t^y) + \beta \log(c_{t+1}^o)$. The measure of agents born in period $t \geq 1$ is equal to $\exp(tg)$, so the total supply of labor in period t is $l_t = \exp(tg)$. In addition, in period $t = 1$, there is a cohort of size 1 of initial old agents who are each endowed with k_1 units of capital. These agents' consumption is denoted c_1^o and their utility by $\log(c_1^o)$. Each period, there are competitive firms that rent capital k at rental rate r_t and hire labor l at wage rate w_t to maximize profits from production with production function $y = Ak_A + Bk_B^\alpha l^{1-\alpha}$ with A and B and α being parameters and $k_A + k_B = k$ being the total capital rented. The firm also is subject to the constraints that $k_A, k_B \geq 0$. Firm profits are

$$Ak_A + Bk_B^\alpha l^{1-\alpha} - r_t(k_A + k_B) - w_t l_t$$

Let R_{t+1} for $t \geq 1$ denote the gross interest rate from period t to period $t+1$ and let δ denote the depreciation rate on physical capital. The budget constraint for the initial old is

$$c_1^o = (r_1 + (1 - \delta))k_1$$

(note that we impose that this as an equality) and that for the cohort born in period $t \geq 1$ is

$$c_t^y + \frac{1}{R_{t+1}} c_{t+1}^o = w_t$$

A feasible allocation in this environment is a collection of sequences

$$\{c_t^o, c_t^y, y_t, k_{At}, k_{Bt}, k_{t+1}, l_t\}_{t=1}^\infty$$

such that, with k_1 given, $l_t = \exp(tg)$, $k_t = k_{At} + k_{Bt}$, $k_{At}, k_{Bt} \geq 0$,

$$\exp((t-1)g)c_t^o + \exp(tg)c_t^y + k_{t+1} = y_t + (1 - \delta)k_t$$

and

$$y_t = Ak_{At} + Bk_{Bt}^\alpha l_t^{1-\alpha}$$

for all t.

Part A: Given values of k_t and l_t, show how to calculate the equilibrium rental rate on capital r_t and wage rate w_t that would induce the firm to choose this level of capital and labor as profit maximizing inputs into production. Under what conditions is the equilibrium rental rate on capital equal to A? What happens to the share of labor compensation in production $(w_t l_t / y_t)$ as the capital labor ratio k_t/l_t goes to infinity

Part B: A competitive equilibrium in this economy is a feasible allocation and a collection of interest rates and factor prices $\{R_{t+1}, r_t, w_t\}_{t=1}^{\infty}$ such that at these prices, $R_{t+1} = r_{t+1} + (1 - \delta)$, the allocation maximizes every agents' utility subject to their budget constraint and maximizes firm profits. Given an initial value of k_1/l_1 (we have $l_1 = 1$), derive a different equation for $\{k_{t+1}/l_{t+1}\}$ in equilibrium. Make sure to keep track of two cases, one in which k_t is large enough so that $r_t = A$ and one in which it is not, so $r_t > A$. This difference equation should involve only parameters and k_t and k_{t+1}. What are the steady-state values of k/l implied by this difference equation given parameters? Is there any parameter configuration such that, in equilibrium, k_t/l_t converges to infinity?

Part C: Now consider a version of this economy in which agents have infinite lives rather than overlapping generations. Specifically, let the representative agent have initial endowment and capital k_1 and $l_t = \exp(tg)$ units of labor in each period $t \geq 1$. Let c_t denote the consumption of this household at date t and let this household have utility

$$\sum_{t=1}^{\infty} \beta^t \log c_t$$

Let the resource constraint for the economy be

$$c_t + k_{t+1} = y_t + (1 - \delta) k_t$$

with

$$y_t = A k_{At} + B k_{Bt}^{\alpha} l_t^{1-\alpha}$$

and $k_{At} + k_{Bt} = k_t$ and both greater than or equal to zero. Under what parameter configurations do we have the capital labor ratio k_t/l_t growing to infinity in the long run and hence the labor share of output shrinking to zero.

Part III

This question is worth **40** points total.

Consider the following three equation monetary model

(0.1) $$i_t - E_t\left[\pi_{t+1}\right] - a\left(E_t\left[y_{t+1}\right] - y_t\right) = \rho + e_t^1$$

(0.2) $$i_t = \lambda\pi_t + \mu\left(y_t - \overline{y}_t\right) + \overline{I}$$

(0.3) $$\pi_t = \beta E_t\left[\pi_{t+1}\right] + \eta\left(y_t - \overline{y}_t\right) + e_t^2$$

In this model i_t is the money interest rate, π_t is the log difference of the price level between periods t and $t-1$, y_t is the log of real GDP, $\overline{y}_t$ is the log of potential output, e_t^1 and e_t^2 are fundamental shocks to demand and supply and $a, \rho, \overline{I}, \beta, \lambda, \mu$ and η are parameters.

A. (2 points) Equation (0.1) is often derived by linearizing the Euler equation of a representative agent. What is the interpretation of the parameters a and ρ?

B. (2 point) If the representative agent had logarithmic preferences, what would that imply for the value of a?

C. (2 points) If you were to estimate this equation and find that ρ was negative, would that be a problem for your interpretation of the equation as an Euler equation? Explain your answer.

D. (2 points) Equation (0.2) represents the policy rule of the central bank. Assume that $\mu = 0$. What is meant by the Taylor principle and what constraints does it place on the remaining coefficients of this equation?

E. (2 points) Much of the new Keynesian literature has attempted to provide a microfoundation to Equation (0.3). Explain **briefly** one possible microfoundation for this equation.

F. (2 points) Assume that the model is well approximated by the parameter restriction $\beta = 1$. What does this imply for the slope of the long-run Phillips curve in this model?

G. (4 points) Assume that $\mu = 0$ and $\overline{y}_t = \overline{y}$ for all t, and retain the assumption that $\beta = 1$. Set the fundamental shocks, e_t^1 and e_t^2 to zero and find expressions for the steady state values of i_t, y_t and π_t in terms of the parameter vector $\theta = \left\{a, \rho, \overline{I}, \lambda, \eta, \overline{y}\right\}$.

H. (1 point) Assume that the Taylor principle holds and that the central bank increases $\overline{I}$. What will be the impact on the steady state value of inflation?

I. (3 points) Assume that the Taylor Principle holds and set $\mu = 0$. Let $\tilde{y}_t$ and $\tilde{\pi}_t$ represent deviations of y_t and π_t from their steady state values. Find a first order vector difference equation in which $x_t = \left[\tilde{y}_t, \tilde{\pi}_t\right]^T$ is written as a function of $E_t\left[x_{t+1}\right]$ and the vector of fundamental shocks.

J. (3 points) Assume that the fundamental shocks have mean zero and are i.i.d. and that $a > 0$ and $\eta > 0$. Find a rational expectations equilibrium of this model by solving the model forwards to express x_t as a function of $e_t = \left[e_t^1, e_t^2\right]^T$.

K. (3 points) Prove that the solution to part J is the unique rational expectations solution if the Taylor principle holds. Hint; assume first that $\lambda = 1$ and show that in this case, the forward looking system has one root of unity and one root that is less than one in absolute value.

L. (3 points) Assume that the Taylor Principle **does not** hold. Prove that the forward looking system from part K has one root outside and one root inside the unit circle.

M. (2 points) In the case that the Taylor Principle does not hold, how many rational expectations equilibria are there in this model?

N. (1 point) What is meant by the "Great Moderation"?

O. (2 points) Briefly sketch the explanation that Clarida, Gali and Gertler gave for the Great Moderation. How is their explanation related to your answers to parts J, K and L?

P. (6 points) Using an eigenvalue decomposition, show how to generate a family of rational expectations equilibria in the case when the Taylor principle fails to hold. How do the dynamics of these equilibria differ from those of the case when the Taylor principle holds and the equilibrium is locally unique?