UCLA Department of Economics **Summer 2011**

PhD. Qualifying Exam in Macroeconomic Theory

Instructions: This exam consists of three parts, each consisting of two questions. You are to choose one question from each part and then a fourth question from any part of your choosing. Answer each question in a separate bluebook. All four questions will receive equal weight in your grade.

Part I

Question 1

Consider an optimal growth model in which the utility of an infinitely lived individual is given by $\sum_{t=0}^{\infty} \beta^t (\log c_t + A \log c_{t-1})$ The population, which is denoted by N_t, is growing over time with growth factor $\eta > 1$. The technology for producing output, *y*, is $y_t = \gamma^t k_t^{\theta} h_t^{1-\theta}$, where $\gamma > 1$. Hours worked, h_t, is constrained to be between 0 and 1. Capital, k_t, depreciates fully each period, so the resource constraint is given by $c_t + k_{t+1} \leq y_t$.

A. Write down the sequence problem that would be solved by a social planner that gives equal weight to individuals of current and future generations.

B. Write the planner's problem from part (A) as a *stationary* dynamic program. Be sure to define any change of variables that you introduce.

C. Use value iteration and the method of undetermined coefficients to solve for the optimal law of motion for the capital stock. Carefully explain each step of your solution procedure.

D. Define a balanced growth path for this economy. Explain what is meant by the term "balanced growth."

E. Suppose you wanted to augment this economy so that leisure is valued. How can you do this in way that is consistent with balanced growth? Explain.

Question 2

Consider the following stochastic optimal growth problem:

$$\max E\sum_{t=0}^{\infty}\beta^t \log c_t + A\log(1-h_t)$$

subject to

$$c_t = e^{z_{1t}}k_{1t}^{\theta}h_{1t}^{1-\theta}$$

$$k_{t+1} = (1-\delta)k_t + e^{z_{2t}}k_{2t}^{\theta}h_{2t}^{1-\theta}$$

$z_{i,t+1} = \rho z_{it} + \varepsilon_{i,t+1}$, for $i = 1, 2$, where $\varepsilon_{i,t+1}$ are independent with mean 0

$$h_{1t} + h_{2t} = h_t$$

$$k_{1t} + k_{2t} = k_t$$

k_0 is given.

A. Decentralize this economy by defining a *recursive competitive equilibrium*. Be clear about the agents that populate your economy and the markets that are open.

B. Derive a set of equations that characterize the *sequence* of prices and quantities that satisfy the definition in part (A) given k_0. Do the quantities that satisfy these equations also characterize the solution to the planner's problem? Explain.

C. Derive log linear approximations to two of the equations derived in part (B). In particular, one of the two equations chosen should involve expectations of the future.

D. Explain carefully how the full set of equations partially derived in part (C) can be solved in a way such that the transversality condition is satisfied.

Part II

Question 3

In this problem, we consider the implications of the standard growth model with infinitely lived agents for the relationship between the savings rate and the growth rate when differences in growth rates are driven by differences in the growth of TFP. We then compare those implications to those of the OLG model.

Part A: Consider a standard growth model in which time is denoted $t = 1, 2, \ldots$, the representative household has preferences of the form

$$\sum_{t=1}^{\infty} \beta^t \frac{1}{1-\sigma} c^{(1-\sigma)},$$

output is given by

$$y_t = a_t k_t^{\alpha},$$

the resource constraint by

$$c_t + k_{t+1} - (1-\delta)k_t = y_t,$$

and in which a_t grows at a constant rate $a_{t+1} = a_t(1+g_a)$. The initial capital stock k_1 is given.

Derive expressions for the relationship between the constant growth rate of a (g_A) and the corresponding balanced growth path growth rates of output y and the savings rate on the balanced growth path as functions of the parameters $g_A, \delta, \beta, \alpha, \sigma$. Give examples of parameter values for which the savings rate does not vary with the balanced growth path growth rate of output g and parameter values for which the savings rate does increase with g. Give an example of other parameter values for which the savings rate decreases with g.

Part B: Now consider an OLG model in which in which time is denoted $t = 1, 2, \ldots$. In each period t an agent is born, and that agent lives for only two periods. Thus, in each period $t \geq 1$, there are two agents alive: a young agent born at t and an old agent born at $t-1$. In period $t = 1$ we assume that there is an initial old agent who owns the initial capital stock k_0.

The agent born in period t has consumption denoted c_t^y and c_{t+1}^o and preferences

$$\log(c_t^y) + \beta \log(c_{t+1}^o).$$

These agents are each endowed with one unit of labor while young and nothing while old. Let output be given by

$$y_t = a_t k_t^{\alpha} l_t^{(1-\alpha)},$$

and the resource constraint by

$$c_t^y + c_t^0 + k_{t+1} - (1-\delta)k_t = y_t.$$

The resource constraint on labor is $l_t = 1$. Assume a_t grows at a constant rate $a_{t+1} = a_t(1+g_a)$.

Derive expressions for the relationship between the constant growth rate of a and the corresponding balanced growth rates of output y and the savings rate on the balanced growth path as functions of the parameters $g_A, \delta, \beta, \alpha, \sigma$. Give examples of parameter values for which the savings rate does not vary with the balanced growth rate of output g.

Part C: Generalize the model in part B to allow preferences of the form

$$\frac{1}{1-\sigma}\left((c_t^y)^{(1-\sigma)} + \beta(c_{t+1}^o)^{(1-\sigma)}\right).$$

Can you find parameter values for which the savings rate does vary with the balanced growth path growth rate of output?

Question 4

In this problem, we consider the implications of a standard labor search model for the relationship between wage dispersion and the observed rate at which agents transit from unemployment to employment.

Consider a standard labor search model in discrete time in which time is denoted $t = 1, 2, \ldots$, and agents have preferences of the form

$$(1-\beta)\sum_{t=1}^{\infty}\beta^t c_t.$$

Agents who have a job that pays wage w earn that wage in the current period and keep that job next period with probability $(1-\delta)$ and lose that job and become unemployed with probability δ. Agents who are unemployed earn an unemployment benefit b and draw a job offer offering constant wage w starting next period, where w is drawn from a distribution with support $[0, \bar{w}]$, cumulative distribution function $F(w)$, and density $f(w)$.

In a stationary equilibrium, we can write the values of having a job with wage w ($W(w)$) and of being unemployed (U) as

$$W(w) = (1-\beta)w + \beta[(1-\delta)W(w) + \delta U]$$

$$U = (1-\beta)b + \beta\int_0^{\bar{w}} \max[W(w), U]dF(w)$$

Part A: Assume that w is uniform on $[0, \bar{w}]$. Compute the reservation wage w^* as a function of the parameters β, b, δ, and $\bar{w}$.

What is the probability that an agent transits from unemployment to employment as a function of these parameters.

Part B: What is the support, the mean, and the variance of the wages for agents who have jobs in equilibrium?

Describe what happens to the variance of the distribution of wages for agents who have jobs in equilibrium and to the probability that an agent transits from unemployment to employment as functions of the parameters β, b, δ, and $\bar{w}$.

Part III

Question 5

Competitive allocation and asset prices. Consider an economy with two infinitely lived consumers indexed by $i = 1, 2$. There is one nonstorable consumption good. Consumer i consumes c_t^i at time t and ranks consumption streams according to

$$\sum_{t=0}^{\infty} \beta^t u(c_t^i),$$

where $\beta \in (0, 1)$ and $u(c)$ is increasing, strictly concave, and twice continuously differentiable. Consumer 1 is endowed with a stream of consumption goods $y_t^i = 1, 0, 0, 0, 1, 0, 0, 0, 1, \ldots.$ Consumer 2 is endowed with a stream of consumption goods $0, 1, 1, 1, 0, 1, 1, 1, 0 \ldots.$ Assume that there are complete markets with time 0 trading.

a) Define a competitive equilibrium.

b) Compute a competitive equilibrium.

c) Suppose that one of the consumers markets a derivative asset that promises to pay 0.025 units of consumption each period, starting at $t = 1$. What would the price of that asset be?

d) Suppose that another consumer markets a derivative asset that promises to pay 10 percent of the endowment of agent 2. What this means is that the derivative pays $0, 0.1, 0.1, 0.1, 0, \ldots.$ What is the price of this derivative at time zero?

e) Consider again the same derivative asset of question d). What would be the price of this derivative at time $t = 3$? More specifically, suppose that once we reach $t = 3$ we sell this derivative before the payment at $t = 3$. How many consumption goods we will receive at time $t = 3$ from this sale?

Question 6

Monopolistic competition. Consider a representative household that maximizes the lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + \log\left(\frac{m_t}{p_t}\right) - \frac{h_t^{1+\varphi}}{1+\varphi} \right]$$

where m_t is the nominal quantity of money and p_t is the nominal price (monetary units). The variable c_t results from the consumption of differentiated goods, that is,

$$c_t = \left(\int_0^1 c_t(i)^{\frac{1}{1+\eta}} \mathrm{d}i \right)^{1+\eta}$$

There is a continuum of firms each producing a differentiated consumption good i with the following production technology:

$$c_t(i) = z_t h_t(i)^{1-\theta}.$$

The nominal price for good i is denoted by $p_t(i)$. Assume that the aggregate supply of money M is fixed.

a) Derive the aggregate price index p_t as a function of individual prices $p_t(i)$.

b) Derive the demand function for each intermediate input. This will be a function of aggregate consumption, c_t, and of the relative price $p_t(i)/p_t$.

c) Write down the optimization problem solved by an individual monopolistic producer and derive the first order condition.

d) Derive expressions for the 'marginal cost' and the 'marginal revenue' for an individual producer as a function of 'individual production'.

e) Assume that households trade the shares of firms at the end of the period after the payment of current profits as dividends. What is the market price of the shares of firm i in a steady state equilibrium?