UCLA Department of Economics

FIRST-YEAR CORE Examination in

MACROECONOMIC THEORY

Spring 2008

This is a 4 hour closed book/closed notes exam.

Instructions:

- **You have 4 hours for the exam.**
- **Answer all 3 questions. All questions are weighted equally.**
- **Use a SEPARATE blue book to answer each question.**
- **Calculators and other electronic devices are not allowed.**

GOOD LUCK!

Macro Comprehensive Exam
Summer 2008

Question 1

Consider a two sector neoclassical growth model in which a nonstorable intermediate good is produced from capital and labor, and final output is produced from the intermediate good and labor. In particular, final output is produced using the technology, $y_t = z_{1t} q_t^{\theta} (\gamma^t h_{1t})^{1-\theta}$, where q_t is the quantity of the intermediate good. The intermediate good is produced using the technology $q_t = z_{2t} k_t^{\lambda} (\gamma^t h_{2t})^{1-\lambda}$. The shocks to technology, z_{1t} and z_{2t}, are governed by the stochastic process, $\log z_{i,t+1} = \rho \log z_{1,t} + \varepsilon_{i,t+1}$ $(i = 1,2)$ where $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$. The parameter $\gamma > 1$ is the labor augmenting growth factor. The intermediate good can only be used in the production of the final good, and final output can be used for consumption or investment (x_t). The capital stock evolves according to $N_{t+1} k_{t+1} = (1-\delta) N_t k_t + N_t x_t$. All quantities are expressed in *per capita* terms. The initial capital stock, k_0, is given.

There are N_t identical individuals at date t and $N_{t+1} = \eta N_t$, where $\eta > 1$ and $N_0 = 1$. Preferences are given by $\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma}$. Each individual has one unit of time that it can allocate between labor and leisure.

A. Write the social planners problem for this economy as a *stationary* dynamic program. Be sure to define any change of variables required.

B. Derive a set of equations (first order conditions and Euler equations) that characterize the dynamic stochastic process for $y_t, q_t, h_{1t}, h_{2t}, k_t$, and c_t.

C. Given a set of parameter values, can the set of equations obtained in part B be used to obtain sequences of realizations for the set of variables listed in part B? Explain. If not, describe a solution procedure that will enable you to obtain such sequences. In particular, if there is a problem obtaining these sequences directly from the equations derived in part B, how do you overcome this difficulty?

D. Characterize (but don't solve for) the balanced growth path for this economy. Be sure that you have the right number of equations as unknowns. Describe the relationship, if any, between the balanced growth path and the solution to the problem in part A.

E. Let $\gamma = \eta = 1$. Define a *recursive competitive equilibrium* for such an economy. Include markets for final output, the intermediate good, labor and capital services.

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Private Information in General Equilibrium

Question 2

Consider the following economic environment. There is measure one of ex-ante identical agents. Ex post, agents turn into one of two types (denoted by θ), productive ($\theta = 1$) and unproductive ($\theta = 0$). The two realizations have equal probability (i.e., each type makes up one-half of the population ex post). The type of each agent is private information. Agents either work or don't work, thus labor supply is $l \in \{0, 1\}$. A worker who doesn't work does not produce anything. A productive worker who works produces four units of output with probability 0.5, and nothing otherwise. Unproductive workers never produce anything, regardless of whether they actually work or not. The realization of output is independent across workers; thus, if x productive workers work, total output is $2x$. Labor input is observable, but an individual worker's output is not.

The utility function of the productive type is:

$$U_{\theta=1} = 2\sqrt{c} - l,$$

whereas the unproductive type has the utility function:

$$U_{\theta=0} = c - l.$$

Each agents is endowed with one unit of the consumption good.

(a) Specify this economy in the language of Debreu. The commodities should be joint lotteries over consumption, labor supply, and type (productive or unproductive). For consumption, you can use the discrete grid:

$$C = \{0, 1, c_{\max}\}.$$

Choose a suitable commodity space, define the consumption and production sets, define the agent's utility function, and write down the resource constraint. Make sure to include incentive constraints that induce truthful reporting of the realized type $\theta \in \{0, 1\}$.

(b) Describe the timing of this economy. When are commodities traded, and when does consumption take place relative to the revelation of information and production?

(c) Formulate and solve the social planning problem for this economy. Specify $c_{\max}$ to be the smallest value that allows the first-best allocation to be achieved. Which incentive constraint is binding? (*Hint: All productive workers should be working in the optimal allocation. Start by determining the optimal allocation of consumption across types given the resulting output, and then consider how lotteries can be used to induce truth telling.*)

(d) Show how the first best can be achieved in equilibrium, i.e., find an equilibrium allocation and price vector. (*Hint: For prices, you can restrict attention to lotteries that are actually traded in equilibrium. Focus on the profit maximization problem to determine equilibrium prices.*)

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Consider an infinite horizon growth model in which agents face idiosyncratic uncertainty regarding their preferences over the timing of their consumption. Number the periods $t = 0, 1, 2, \ldots$. Each period, for each consumer, an event θ_t is realized with $\theta_t \in \Theta = \left\{\theta^1, \ldots, \theta^N\right\}$. These events are i.i.d. across time and across consumers with probabilities $\rho(\theta^n)$. We assume that $\rho(\theta^n)$ also represents the fraction of agents who receive shock θ^n. Let $h^t = (\theta_0, \theta_1, \ldots, \theta_t)$ denote the history of events an agent has experienced through date t and let $\pi_t(h^t)$ denote the date zero probability of this history. Let $\theta_t(h^t)$ denote the last element of history h^t.

An allocation in this economy is a sequence of consumption plans and physical capital stocks

$$\left\{c_t(h^t), k_{t+1}\right\}_{t=0}^{\infty}$$

with the initial capital stock k_0 given.

The resource constraints in this economy are that

$$\sum_{h^t \in \Theta^t} c_t(h^t)\pi_t(h^t) + k_{t+1} = f(k_t)$$

for all t, with $f(k) = k^\alpha$ with $\alpha \in (0, 1)$.

Consumers have preferences given by

$$\sum_{t=0}^{\infty} \beta^t \sum_{h^t \in \Theta^t} \theta_t(h^t) \log(c_t(h^t))\pi_t(h^t)$$

a) Full Information Social Optimum part 1

Write down the first order conditions characterizing the allocation that maximizes the utilitarian social welfare function

$$\sum_{h^t \in \Theta^t} c_t(h^t)\pi_t(h^t) + k_{t+1} = f(k_t)$$

subject to the resource constraints. Solve for the steady-state capital stock in this optimal allocation. Does your solution for the steady-state capital stock here depend on the distribution of shocks θ?

b) Full Information Social Optimum part 2

Show that the sequence of capital stocks and aggregate consumption

$$c_t = \sum_{h^t \in \Theta^t} c_t(h^t)\pi_t(h^t)$$

in the solution of the full information social optimum are the same as those found from a representative agent economy in which the representative agent has preferences of the form

$$\sum_{t=0}^{\infty} \beta^t \log(c_t)$$

and the resource constraints are given by

$$c_t + k_{t+1} = f(k_t)$$

for all t, with $f(k) = k^\alpha$ with $\alpha \in (0,1)$.

Can we get the same result with more general period utility functions of the form $u(c)$ other than $\log(c)$?

Use this result to solve for the entire allocation that solves the full information social planning problem with log utility.

c) Bond economy

Consider now an incomplete markets economy in which, at each date t, agents only trade shares of the aggregate capital stock. Let q_t denote the price of this share and $s_t(h^t)$ the quantity purchased by an agent with history h^t. Given this notation, agents have budget constraints

$$c_t(h^t) + q_t\left(s_t(h^t) - s_{t-1}(h^{t-1})\right) = s_{t-1}(h^{t-1})k_t^\alpha$$

for all $t > 0$ and h^t. At date 0, assume that agents all start with initial shareholdings $s_{-1} = 1k_0^\alpha$ so they face constraint

$$c_0(h^0) + q_0(s_0(h^0) - 1) = k_0^\alpha$$

Agents are constrained to have non-negative shareholdings $s_t(h^t) \geq 0$.

The consumers' in this economy choose consumption $\{c_t(h^t)\}_{t=0}^\infty$ and shareholdings $\{s_t(h^t)\}_{t=0}^\infty$ to maximize their utility

$$\sum_{t=0}^{\infty} \beta^t \sum_{h^t \in \Theta^t} \theta_t(h^t) \log(c_t(h^t))\pi_t(h^t)$$

subject to their budget constraints.

Define (gross) interest rates R_t by

$$R_t = \frac{k_{t+1}^\alpha + q_{t+1}}{q_t}$$

A representative firm in this economy takes share prices and interest rates $\{q_t\}_{t=0}^\infty$ and interest rates $\{R_t\}_{t=0}^\infty$ as given as well as the initial capital stock k_0 and chooses a sequence for capital $\{k_t\}_{t=1}^\infty$ to maximize the discounted present value of its dividends

$$\sum_{t=0}^{\infty} \prod_{s=0}^{t} R_s^{-1}\left[k_t^\alpha - k_{t+1}\right].$$

An equilibrium in this economy is a sequence of share prices and interest rates $\{q_t, R_t\}_{t=0}^{\infty}$, shareholdings $\{s_t(h^t)\}_{t=0}^{\infty}$ and an allocation $\{c_t(h^t), k_{t+1}\}_{t=0}^{\infty}$ such that agents are maximizing their utility given prices, firms are maximizing the discounted present value of dividends, and the resource constraints are all satisfied.

Describe how to solve for the steady-state capital stock in this economy.

d) Bond Economy Part 2

Define $\bar{\theta} = \sum_{n=1}^{N} \theta^n \rho(\theta^n)$.

Show that the equilibrium consumption plan has the form

$$c_t(h^t) = \frac{\theta_t(h^t)(1-\beta)}{\bar{\theta}\beta + \theta_t(h^t)(1-\beta)} s_{t-1}(h^{t-1})(k_t^{\alpha} + q_t).$$

Find the equilibrium values of q_t as functions of capital k_t.

Is the steady-state value of capital in this bond economy higher or lower or the same as that in the full information social optimum?