

**Macroeconomic Theory
Ph.D. Qualifying Examination
UCLA Department of Economics**

July 2006

Instructions: You have 4 hours to complete the exam. Answer all six questions. Each question has equal weight. **Answer each question in a separate blue book.**

Warning: You may be under time pressure for this exam. Budget your time accordingly and do not waste too much time on any one question. The average time for each question is 40 minutes.

Question 1

An economic model gives rise to the following set of equations.

$$Y_t = K_t^\alpha (A_t)^{1-\alpha}, \quad (1)$$

$$K_{t+1} = K_t(1 - \delta) + Y_t - C_t, \quad (2)$$

$$A_t = (1 + g) A_{t-1} \exp(e_t), \quad (3)$$

$$\frac{1}{C_t} = E_t \left\{ \frac{\beta}{C_{t+1}} \left(1 - \delta + \alpha \frac{Y_{t+1}}{K_{t+1}} \right) \right\}. \quad (4)$$

Y_t is output, C_t is consumption K_t is capital and A_t is an exogenous productivity shock. The parameters δ, α, g and β are all between 0 and 1. e_t is a random variable with zero expected value.

1. Define a set of transformed variables, and write down a set of equations in these variables, such that a balanced growth path for the original model can be represented as a stationary state of the transformed model. **(2 points)**
2. Find explicit expressions for the steady state values of the transformed variables. **(2 points)**
3. Compute the parameters of the first order log-linear approximation to this model and, using the method described by Chris Sims in the paper "Solving Linear Rational Expectations Models" write down a linear matrix difference equation that describes the behavior of deviations of the model variables from their steady state values. **(3 points)**
4. Explain in words how to compute a solution to the model using the QZ decomposition. **(3 points)**

Question 2

This question refers to the two-person one-good overlapping generations model with a single type of agent in each generation. Preferences of the representative agent are given by the equation

$$U_t = E_t \left[\frac{(c_{t+1}^t)^{1-\rho}}{1-\rho} \right] - \frac{n_t^2}{2}, \quad (5)$$

where U_t is utility of generation t , c_s^t is consumption of generation t at date s and n_t is labor supply at date t . E_t is the expectations operator conditional on date t information. Output is equal to the labor supply of the young

$$y_t = n_t. \quad (6)$$

The initial old generation is endowed with a single unit of fiat money and has utility

$$U_1 = c_0^1.$$

All output is consumed by the older generation in each period. There is no fundamental uncertainty.

1. Write down a stochastic difference equation, using n_t as a state variable, that must be satisfied by competitive equilibrium allocations. (5 points)
2. For what values of ρ does this model possess sunspot equilibria? Explain your answer. (5 points)

Question 3

The maximization problem for a representative household is:

$$\max \sum_{t=0}^{\infty} \beta^t \{ \alpha \ln(c_{1t}) + \phi \ln(1 - l_t) \},$$

The resource constraints for the consumption and investment goods are:

$$\begin{aligned} C_t &\leq A_1 K_{1t}^\theta (X_{1t} L_{1t})^\nu, \quad 0 < \nu < 1 - \theta, \\ I_t &\leq A_2 K_{2t}^\theta (X_{2t} L_{2t})^{1-\theta}, \end{aligned}$$

The law of motion for the capital stock is given by:

$$K_{t+1} = I_t + (1 - \delta)K_t,$$

The remaining constraints are given by:

$$\begin{aligned} K_t &= K_{1t} + K_{2t}, \\ X_{1t} &= (1 + \gamma_1)^t, \quad X_{2t} = (1 + \gamma_2)^t, \quad \gamma_1 > 0, \gamma_2 > 0. \end{aligned}$$

1. Define a stationary recursive equilibrium for this economy, and let lower case letters be stationary variables. Let the numeraire be new investment, I . Denote the price of C as p_1 , denote the price of capital, K , as p_2 . Explain why I and K sell for the same price. Present a formula for the relative price of consumption in the stationary economy and in the non-stationary economy. Explain why the size of the parameter ν helps determine the relative price of consumption. Explain (in words) why the competitive equilibrium allocations and the quantities chosen by a benevolent social planner coincide. **(6 points)**
2. Suppose that the economy is in a steady state, and ν jumps to the value $1 - \theta$. What should happen to the values of stationary steady state K, C, p ? What happens to GDP, measured as $Y_1 + pY_2$. Explain your answers. Sketch what you think the time paths of C, I, p , and L will be on the transition to the new steady state. (Note there is no closed form solution, but you should be able to graph time paths for the variables and explain the economic forces generating these paths between the old and new steady states). **(4 points)**

Question 4

Consider an economy extending over two periods, $t = 1, 2$, which is populated by a continuum of ex-ante identical households of measure one, indexed by i . Preferences are given by:

$$E \left[\sqrt{c_{i,1}(1 - n_{i,1})} + \beta \sqrt{c_{i,2}(1 - n_{i,2})} \right]$$

where $c_{i,t}$ is consumption, and $n_{i,t}$ is labor supply. During period $t = 1$, which corresponds to education, people enter one of two professions, business or economics. Business does not require education, hence $n_{i,1} = 0$ for a businessperson, while economics requires a Ph.D., hence $n_{i,1} = 0.75$ for an economist. During the second period, members of both occupations work $n_{2,i} = 0.5$. During the first period, consumption is derived from an aggregate endowment of $R = 1$. The consumption good cannot be stored. During the second period, the consumption good is produced using a production function given by:

$$F(B, E) = \sqrt{BE},$$

where B is the mass of businesspeople and E is the mass of economists. Given that there is mass one of people and everybody has to choose an occupation, we must have $B + E = 1$.

1. Specify this economy in the language of Debreu, i.e., choose a suitable commodity space, define the consumption and production possibility sets, define the utility function, and specify the resource constraint. Make sure to use a lottery to deal with the indivisible occupational choice (people can either go into business or economics, but they cannot do both jobs part time). **(5 points)**
2. In equilibrium, are there going to be more economists or more businesspeople? Which group is going to have higher consumption in period 1, which one will have higher consumption in period 2? (No formal analysis required—answer with short intuitive justification suffices). **(5 points)**

Question 5

Assume that we have a unit measure of households, and that each household receives a stochastic income stream $\{y_t\}_{t=0}^{\infty}$, and has preferences given by

$$E \sum_{t=0}^{\infty} \beta^t u(c_t).$$

A household's income in each period is an i.i.d. draw from the set $Y = \{y_1, \dots, y_S\}$, where $y_s < y_{s+1}$, and $y_1 > 0$. Let Π_s denote the probability of receiving income y_s . We will denote the household's history by

$$h_t = \{y_0, y_1, \dots, y_t\}.$$

We will assume that these households enter into a financial contract with an agent who can borrow and save at a fixed per-period interest rate $R = \beta^{-1}$, and therefore has objective function

$$E \sum_{t=0}^{\infty} \beta^t (y_t - c_t). \quad (7)$$

The households always have the option of walking away from their contract with the financial intermediary. A contract is a sequence of functions $\{f_t\}$:

$$c_t = f_t(h_t).$$

Define the expected autarky payoff as:

$$v_{aut} = E \sum_{t=0}^{\infty} \beta^t u(y_t).$$

Assume that the enforcement constraint is given by

$$u[f_t(h_t)] + \beta E_t \sum_{j=1}^{\infty} \beta^{j-1} u[f_{t+j}(h_{t+j})] \geq u(y_t) + \beta v_{aut} \quad (8)$$

1. Set up the recursive social planning problem and construct the first-order conditions. **(5 points)**
2. Under what circumstances does an individual's enforcement constraint bind? **(5 points)**

Question 6

Consider a standard neo-classical production economy with a single consumption good.

Household: The representative household maximizes

$$\sum_{t=0}^{\infty} \beta^t [U(c_t) - v(L_t)],$$

where c_t denotes consumption as of date t , and L_t the household's labor supply. U is increasing and concave, and v is increasing and convex, with $\lim_{c \rightarrow 0} U'(c) = \infty$ and $\lim_{L \rightarrow 0} v'(L) = 0$. The household accumulates capital, which depreciates at a rate $\delta \in (0, 1)$.

Firms: In each period, there is a large number of firms, which use capital k_t and labor l_t to produce output, using a technology $y_t = f(k_t, l_t)$ which has constant returns to scale.

1. Set up the social planner's problem for this economy. Taking first-order conditions, provide a set of conditions that characterize the optimal capital stock and labor supply in steady-state. **(3 points)**

Consider now the following Cash-in-Advance economy. At the beginning of each period, the household receives a monetary lump sum transfer (or tax) of T_t . Then, markets open. The household must use money balances after the transfer to purchase consumption and investment. It supplies labor services and rents out capital to the firms, and collects the resulting revenues only at the end of each period.

2. Set up the representative household's dynamic optimization problem. Make sure to properly specify the household's budget constraint and the Cash-in-Advance constraint, and define the competitive equilibrium of this cash-in-advance economy. Be sure to derive all relevant optimality and market-clearing conditions that must hold in equilibrium. **(4 points)**
3. Suppose now that the money injections take the form $T_t = \mu M_{t-1}$, where $\mu \geq \beta - 1$ denotes the growth rate of money. Characterize the steady-state conditions for the Cash-in-Advance economy. Does this economy achieve the efficient allocations of investment consumption and labor as characterized under 1 for the social planner's problem? If so, why? If not, why not? **(3 points)**