Macroeconomic Theory

Ph.D. Qualifying Examination
July 2005

Comprehensive Examination **UCLA Dept. of Economics**

- **You have 4 hours to complete the exam.**
- **There are three parts to the exam.**
- **Answer all parts. Each part has equal weight.**

ANSWER EACH PART IN A SEPARATE BLUE BOOK.

PART ONE: ANSWER IN BOOK 1
WEIGHT 1/3

Answer BOTH questions

1. Consider the following real business cycle model (social planning problem). There is a continuum of individuals, and the planner makes choices in order to maximize, $E\sum_{t=0}^{\infty}\beta^t\left[n_t U(c_{1t},1-\phi-e_t\bar{h})+(1-n_t)U(c_{2t},1)\right]$. Here, n_t is the fraction of individuals that work $\bar{h}$ in period t, c_{1t} is consumption of an individual who works in date t, and c_{2t} is consumption of an individual who is not assigned to work in period t. The variable e_t is effort and reflects how hard an individual is required to work and ϕ is a parameter reflecting time spent commuting.

The planner maximizes this objective subject to a resource constraint. There is one good that is produced using a constant returns to scale technology with capital and labor, $y_t = z_t F(k_t, n_t e_t \bar{h})$. Capital, k_t, evolves over time according to $k_{t+1}=(1-\delta)k_t + x_t$, where x_t is gross investment. The variable z_t is a technology shock realized at the beginning of period t. The stochastic process for this shock is a two-state Markov chain with state space $\{z_L, z_H\}$ and transition matrix P.

The planner must choose n_t before observing the current shock, z_t (say at the end of the previous period). All other date t variables (except k_t) are chosen after observing z_t.

Formulate this planner's problem as a dynamic program. Be careful to insure that your dynamic program is well defined.

2. Consider the following stochastic optimal growth problem:

$$\max E\sum_{t=0}^{\infty}\beta^t U(c_t, 1-h_t)$$

subject to

$$c_t = e^{z_{1t}}F(k_{1t}, h_{1t})$$

$$k_{t+1} = (1-\delta)k_t + e^{z_{2t}}G(k_{2t}, h_{2t})$$

$$z_{i,t+1} = \rho z_{it} + \varepsilon_{i,t+1}, \text{ for } i = 1,2,$$

$\varepsilon_{i,t+1}$ are independent iid random variables with mean 0

$$h_{1t} + h_{2t} = h_t$$

$$k_{1t} + k_{2t} = k_t$$

k_0 is given.

The functions F and G are standard concave, continuously differentiable, constant returns to scale production functions. The utility function is also concave and continuously differentiable. The z_i's are random variables observed at the beginning of the period and capital and labor can be freely moved across sectors.

A. Write this planner's problem as a dynamic program. Make sure that your dynamic program is well defined.

B. Decentralize this economy by defining a *recursive competitive equilibrium*. Be extremely clear about the agents that populate your economy and the markets that are open.

C. Show that the set of equation characterizing a solution to a the planner's problem in part A also characterize the competitive equilibrium defined in part B (you can ignore transversality conditions).

PART TWO: ANSWER IN BOOK 2
WEIGHT 1/3

Answer all parts

Consider the following linear rational expectations model which describes the behavior of a scalar variable p_t for $t = 1,..\infty$

(1.1) $$p_t = \frac{1}{\lambda+\theta} E_t\left[p_{t+1} | \Omega_t\right] + \frac{\lambda\theta}{\lambda+\theta} p_{t-1} + v_t,$$

(1.2) $$p_0 = \overline{p}_0,$$

where v_t is an i.i.d. random variable with density $h(x)$ and support $[-b,b]$, $E_t\left[\bullet | \Omega_t\right]$ is the expectation operator conditional on information set Ω_t and λ and θ are roots of the characteristic polynomial of (1.1), i.e.

(1.3) $$x^2 - (\lambda+\theta)x + \lambda\theta = 0.$$

Answer the following questions

A. Using the following definition of Y_t

(1.4) $$Y_t = \begin{bmatrix} p_t \\ E_t\left[p_{t+1} | \Omega_t\right] \end{bmatrix},$$

show how to write Equation (1.1) in the form

(1.5) $$AY_t = BY_{t-1} + \psi_v v_t + \psi_w w_t$$

by defining matrices A, B, vectors ψ_v, ψ_w and a new endogenous error w_t. Your answer should give the elements of A, B and ψ_v, ψ_w and define the term w_t in terms of the parameters λ, θ and the terms p_t and $E_t\left[p_{t+1}\right]$.

B. Give a definition of the generalized eigenvalues of a pair of matrices A, B and explain what is meant by a QZ decomposition.

C. Using your answer to part A, find an expression for the matrix $A^{-1}B$. Using the fact that the roots of $A^{-1}B$ are equal to θ and λ, decompose this matrix into the form

$$A^{-1}B = Q\Lambda Q^{-1}$$

where $\Lambda = \begin{bmatrix} \lambda & 0 \\ 0 & \theta \end{bmatrix}$ and $Q = \begin{bmatrix} 1 & 1 \\ x & y \end{bmatrix}$. Find explicit expressions for x, y and the elements of Q^{-1} in terms of λ and θ.

D. Using your answers to parts A-C show how to write the system as a pair of scalar difference equations

(1.6) $$z_{1t} = \theta z_{1t-1} + \varepsilon_{1t} + \eta_{1t},$$

(1.7) $$z_{2t} = \lambda z_{2t-1} + \varepsilon_{2t} + \eta_{2t},$$

where

(1.8) $$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} = Q^{-1}A^{-1}\psi_v v_t,$$

(1.9) $$\begin{bmatrix} \eta_{1t} \\ \eta_{2t} \end{bmatrix} = Q^{-1}A^{-1}\psi_w w_t,$$

and

(1.10) $$\begin{bmatrix} z_{1t} \\ z_{2t} \end{bmatrix} = Q^{-1}\begin{bmatrix} p_t \\ E_t\left[p_{t+1}|\Omega_t\right] \end{bmatrix}.$$

In your answer you should find explicit expressions for the variables $\varepsilon_{1t}, \varepsilon_{2t}, \eta_{1t}$ and η_{2t} in terms of v_t, w_t and the parameters λ and θ and explicit expressions for z_{1t} and z_{2t} in terms of λ, θ, p_t and $E_t\left[p_{t+1}|\Omega_t\right]$.

E. Assume now that $|\theta| > 1$, $|\lambda| < 1$. Find an explicit expression for the non-fundamental shock η_{1t} as a function of the fundamental shock ε_{1t}.

F. Find an explicit expression for the solution to the rational expectations model (1.1) in the form

(1.11) $$p_t = \alpha p_{t-1} + \beta v_t$$

What are the values of α and β in terms of λ and θ?

G. Explain how the solution to this model would be different if both roots, λ and θ, were less than one in absolute value.

H. What is meant by a Markov process? Does (1.11) define a Markov process?

I. What method could you use to prove that the sequence of random variables $\{p_t\}_{t=1}^{\infty}$ converges to a limiting distribution? What is the support of this distribution?

PART THREE: ANSWER IN BOOK 3
WEIGHT 1/3

Answer ONE question

EITHER

1. Consider the following wage-setting model. Assume that there is a continuum of households indexed by $i \in [0,1]$. Each household supplies a particular type of labor. At the beginning of each period, each household sets the wage $w(i)$ at which they will sell their labor. They do so prior to seeing the realization of any current shocks $s = (\tau, z)$, where τ is the current monetary injection and z is the current productivity level. For simplicity, we will assume that the household is then required to supply whatever labor the firms demand at that wage. We will assume that each household owns a per capital share of the profits of the firms, but for simplicity we will assume that this claim cannot be traded. We will assume that the only traded assets are state-contingent bonds. We will assume that any monetary injections are taxed away between periods to generate a stationary equilibrium outcome.

There is a representative firm which takes the wage schedule as given, and produces output via the different kinds of labor so as to maximize their profits. The production function of the firm is given by

$$z\left(\left[\int_0^1 n(i)^\theta \, di\right]^{1/\theta}\right)^{1-\gamma}$$

where $n(i)$ denotes their demand for each of the different types of labor, given the productivity shock z and the wage schedule $w(i)$.

The households all have identical preferences given by

$$E\sum_{t=0}^{\infty} \beta^t \left(u(c_t) + v(1-l_t)\right)$$

where c denotes consumption of the single final good, and l denotes labor supplied to market. The households's budget constraint and cash-in-advance constraint are given by

$$p(s)c(s) + m' + \int_{s'} b'(s')\,ds' \leq m + R(s)b(s) + w(i)N\left(w(i), w(\bullet), s\right) + \Pi_t + \tau,$$

$$m + \tau \geq p(s)c(s).$$

where N denotes the demand function for the household's labor given his wage, the overall wage schedule and the state, and denotes the household's claim to the profits of the firm. Define the market clearing conditions as

$$c(s) = y(s),$$
$$b'(s) = 0,$$
$$m'(s) = 1 + \tau.$$

A. Denoting the aggregate amount of labor hired by the representative firm by

$$n = \left[\int_0^1 n(i)^\theta \, di \right]^{1/\theta}$$

set up the firm's problem, and use it to derive the demand schedule for the labor supplied by an individual household, N. (It's easier to do this simply as a function of the individual's wage, the aggregate level of labor n and the productivity shock z. But, if you're feeling bold, try to express it as a function of the wage schedule instead of n.)

B. The households have a two stage problem. At the beginning of the period, prior to knowing the current state, they must choose $w(i)$. Then, later in the period after seeing the state s, and having $w(i)n(i)$ determined, they must choose how much to consume c and how much to save via the risk-free state contingent bonds $b(s')$. Construct a stationary recursive representation of the household's two stage problem. Use this problem to derive an expression for their optimal wage setting condition and their optimal holdings of the state-contingent bonds.

C. Assuming that the state is distributed according to $h(s)$, that is, i.i.d., use your optimality conditions from the household's and the firm's problems, along with your market clearing conditions to characterize the equilibrium levels of output, labor effort and the wage rate. To get a nice final set of expression try log-linearizing. Discuss the implications of your results for the effects of monetary and real shocks in this model as compared to the standard cash-in-advance constraint model.

OR

2. Consider the following version of a Ramsey tax model in which we assume that the government can only tax labor income. The rest of the model is standard. Government expenditure is taken to be an exogenous sequence $\{g(t)\}$. In particular, it has the following features:

Government's budget constraint:

$$g_t + (1 + r_t) b_t = b_{t+1} + \tau_{l,t} w_t l_t$$

Household's problem

$$\max \sum_t \beta^t u(c_t, l_t)$$

subject to

$$(1-\tau_{l,t}) w_t l_t + q_t k_t + (1+r_t) b_t = c_t + k_{t+1} - (1-\delta) k_t + b_{t+1}$$

$$k_{t+1} \geq 0$$

$$k_0, b_0, r_0 \text{ given}$$

Firm's Problem:

$$\max\left[f(k_t, l_t) - q_t k_t - w_t l_t \right]$$

Resource Constraint:

$$c_t + k_{t+1} - (1-\delta) k_t + g_t = f(k_t, l_t)$$

A. Construct the Ramsey problem in terms of the allocation variables. Be sure to show how any allocation that satisfies these constraint can be mapped into a competitive equilibrium and associated government policy sequence $\{\tau_{l,t}\}$.

B. Characterize the solution to the Ramsey problem. Assume for the rest of this problem that $g_t \to g$ and the allocation that solves this problem converges to a steady state. Is this steady state any different than the one that would arise if there were taxes on capital? Will labor taxes be zero in the steady state?

C. Assume that g follows a deterministic sequence of high and low levels: with $g_t = g_l$ for t odd and $g_t = g_h$ for t even, where $g_h > g_l$. If the economy settles down into a deterministic sequence with odd period allocations being the same, and even period allocations being the same, can you say anything about the pattern of labor taxes, government deficits and the government's debt?