Macro Comp Spring 2003

You have four hours. The exam is closed book, closed notes. There are four questions which carry equal weight. Please answer all questions. If you find a question to be unclear or you think that additional assumptions are needed, state clearly which assumptions you are making.

Financial Markets and Growth

Question 1

This question explores how the state of financial markets influences the behavior of growth rates in per capita output. Consider an economy consisting of two infinitely-lived households, indexed by $i = 1$ and 2, with common utility function:

$$u_i = \sum_{t=0}^{\infty} \beta^t \log(c_t^i) \qquad 0 < \beta < 1$$

that values infinite consumption streams (c_t^i). Each household owns an initial amount of capital $k_0^i > 0$, has zero initial debts, and a technology

$$y_t^i = z_t^i k_t^i$$

for converting physical capital, k_t, at the start of period t into output y_t in period t. Physical capital is the households' only source of wealth and the only asset of this economy in positive net supply. There are *no* labor endowments or other inputs. You should interpret y_t to include all undepreciated capital. Capital productivities (z_t^1, z_t^2) are positive deterministic sequences of alternating numbers

$$
\begin{aligned}
(z_t^1, z_t^2) &= (zA, A) & &\text{if } t = 0, 2, 4, \ldots \\
&= (A, zA) & &\text{if } t = 1, 3, 5, \ldots
\end{aligned}
$$

where the numbers (z, A) satisfy

$$0 \leq z \leq 1 \qquad \beta A > 1$$

Thus, agent 1 is unproductive in even periods and productive in odd ones while agent 2 has the reverse productivity pattern.

(a) Suppose at first that financial markets are complete: lending moves capital to the most productive technology. Derive the equilibrium yield on loans or physical capital and the equilibrium growth rate of output in terms of the parameters (A, β, z). [Hint: Describe first how each household divides current resources between consumption and saving].

(b) Explain why the output growth rate does not fluctuate in this economy.

(c) Suppose that financial markets are *non-existent*: capital cannot move at all between agents or sectors either by lending or by stock market purchases. Describe the evolution of output in each sector separately, then combine sectoral outputs to characterize the growth rate of total output as a function of the parameters (A, β, z) and the initial distribution of capital. [Hint: use as a state variable the share of capital in the productive sector].

(d) Show that the non-existence of financial markets reduces the economy-wide average growth rate and increases short-term growth rate fluctuations relative to complete financial markets. Explain the economic intuition for these results.

An *AK* Model

Question 2

(a) Consider an economy with a representative consumer whose preferences are defined by the utility function:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\sigma}}{1-\sigma},$$

where $\sigma > 0$ and $0 < \beta < 1$. There is a competitive industry which operates the production technology:

$$y_t = Ak_t,$$

where $A > 0$. Capital evolves according to:

$$k_{t+1} = i_t,$$

where i_t is investment (capital depreciates completely in a period). The initial level of capital is given by k_0, and consumption is restricted to be nonnegative.

- Provide a Bellman equation for the problem solved by a benevolent social planner.

- Define a sequence-of-markets equilibrium for this economy.

- Define a balanced growth path, and find the balanced growth rate. Is the balanced growth path reached instantaneously?

(b) Now assume that the consumer exhibits habit formation and has preferences:

$$\sum_{t=0}^{\infty} \beta^t \frac{(c_t - \phi c_{t-1})^{1-\sigma}}{1-\sigma},$$

where $0 < \phi < 1$.

- Provide a Bellman equation for the social planning problem in this variant of the economy.

- Find the balanced growth rate. Is the balanced growth path reached instantaneously? Explain the economic intuition for your result.

- Is there a balanced growth path for all parameter combinations? If not, provide a condition on the parameters that guarantees existence of a balanced growth path.

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Taxation and Asset Pricing

Question 3

Consider a representative agent economy. The agent has recursive utility

$$U_t = \left((1-\beta) C_t^{1-\frac{1}{\psi}} + \beta \left(E_t U_{t+1}^{1-\gamma} \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right)^{\frac{1}{1-\frac{1}{\psi}}} .$$

The growth rate of the aggregate endowment is normal with mean μ and standard deviation σ. Only one asset is in positive net supply: the claim to the aggregate endowment.

There is a government that runs a balanced budget. It finances government spending g_t using a 20% capital gains tax every period. This tax is levied on all returns the agent gets.

(a) Define an equilibrium for this economy.

(b) Calculate the price dividend ratio and the riskless interest rate in closed form as a function of preference parameters and the tax rate.

(c) Calibrate the model. Use $\mu = .02, \sigma = .01$ and choose preference parameters to match a price dividend ratio of 20 and a riskless interest rate of 1%.

(d) Calculate the effect on the price dividend ratio if the tax rate is reduced to 10%.

(e) Suppose now the government budget is not balanced, but instead that government spending g_t is given exogenously and debt is used to make up the difference between g and taxes. Explain briefly why this would complicate the analysis and a closed form solution would no longer be available.

Greek Politics, Inflation and Investment

Question 4

Premier Azariadis has been running Greece with a policy of 30% inflation, with the government using those revenues to finance investment. His opponent, Athanasios Bolmatis, a hard line free market economist, has argued that this policy is inefficient, and promises to bring inflation to 0, with no government-financed investment. To assess these policies, assume that preferences for the representative consumer are given by:

$$\max \sum_{t=0}^{\infty} \beta^t [\log(c_t) + \phi \log(1 - n_t)]$$

The household faces a CIA constraint:

$$m_{t-1}/p_t \geq c_t$$

The technology for representative firm i is given by:

$$K_t^{\eta} k_{it}^{\theta} n_{it}^{1-\theta}, \qquad 0 < \eta < 1 - \theta$$

(In this technology, lower case variables are firm choices, and upper case variables are taken to be exogenous by the firm.) The resource constraint is given by:

$$Y_t = C_t + I_{pt} + G_t$$

The law of motion for the aggregate capital stock is given by:

$$K_{t+1} = (1 - \delta)K_t + I_{pt} + G_t,$$

where G_t is government spending, which is used entirely for investment, and I_{pt} is private sector investment. Government-financed investment is given to households via lump sum transfers.

The government budget constraint and monetary policy are given by:

$$(M_t - M_{t-1})/p_t = G_t$$

$$M_t = (1 + \mu_t)M_{t-1}$$

(a) Derive an expression for aggregate output. (Given constant returns to scale, assume that there is a single location for production.)

(a) Do the welfare theorems hold for this economy? If not, why not? Define a competitive equilibrium for this economy, and solve for the first order necessary conditions for the firm and the households.

(c) Consider 2 policies: Bolmatis: ($\mu = 0$) and Azariadis: ($\mu = .30$).Derive an expression for steady state objects in which you could solve for steady state welfare differences between the 2 policies. Describe how the size of η will be important for understanding which policy achieves higher steady state welfare. In particular, can you show explicitly how η enters into this welfare calculation?